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ON THE MOTION OF BED-LOAD SEDIMENT AS A RANDOM PROCESS

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ABSTRACT

By treating the motion of bed-load sediment as a random process, several families of functions for the one-dimensional distribution of characteristic phenomena are given.

This analysis is confirmed by experiments done by the authors within the framework of a research project funded by the International Atomic Energy Agency.

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INTRODUCTION

The motion of bed-load sediment is an important problem for the regularization of waterways and the building of hydraulic works. Notwithstanding the research carried out during several decades, one has to conclude that a general theory of the bed-load mechanism is not yet available.

It is generally admitted now that the hydrodynamic forces developed by the contact of the fluid with grains lying on the bed of the stream surmount the resistances due to contact of the grains with the bed, thus resulting in the pull-out of the grains. This pulling-out is only possible in the upper layer. During the process the grains start to vibrate, to gyrate in place and then separate suddenly, losing contact with the bed; by sliding, rolling or jumping they continue the motion, until they are stopped by other grains, or until the instantaneous hydraulic forces become insufficient for the continuation of the motion. This discrete motion of individual grains results into the development of a steady motion of the sediment.

This concise description makes it clear that we have here simultaneous motion of two different physical phases and their reciprocal influences, and, therefore, the essential elements of the process are the structure of the fluid flow, the characteristics of the material of the bed and their contact. The instantaneous values of the hydrodynamic forces vary with time, as well as does the exposure of the grains to hydrodynamic influences, because in time some grains shall be eroded or deposited around a given grain. There is no doubt that all this defines a random process -instantaneous hydrodynamic forces dependent and, consequently, dependent on the turbulent characteristics of the flow and on the instantaneous positions of the grains on the bed.

RANDOM PHENOMENA CONSIDERED

In a historical review of the analyses of the motion of bed-load sediment as a random process, we point out that a longitudinal dispersion of the grain groups was considered first - by Einstein (1937), Lean and Crickmore (1962).

Hubbell and Sayre (1964) have proposed to observe the displacement of individual grains and to analyze the distribution function of the displacement length of a grain and that for a rest period. Grigg (1970), Yang and Sayre (1971) and Stelzer (1972) performed studies based on that idea.

The basic idea of this paper is to analyse the random phenomena that can be experimentally observed, to derive analytic expressions for the distribution functions of the random phenomena considered, and to explain, with the help of such functions, the nature of the motion of the sediment and the main variables which influence its transport and dispersion.

On considering the motion of a grain one can distinguish the periods of displacement and periods of rest, which alternate.

By observing the motion of a single grain, one notes and analyses the following random phenomena:

- the number of stoppings of the grain during a fixed time interval,
- the number of stoppings of the grain during a fixed length interval,
- the duration of the period covering ν cycles of the grain motion (ν displacements and ν resting periods),

- the length of the total trajectory followed during n displacements of the grain.

The above indicated phenomena must be explained by the four families of distribution functions.

On observing the number of stoppings of the grain in the fixed time interval (0,t*), the experiment must be repeated several times to obtain the distribution function . One trial gives an integer as the result; consequently this phenomena can be described but by a discrete distribution function. Similar conclusions are equally valid in observing the number of stoppings of the grain in the fixed length interval (0,x*). One can vary the values of t* and x* by different intervals - for example, t* = 5, 10....60min and x* = 20, 30,....200cm - in the laboratory, and thus obtain two families of discrete distribution functions.

It is important to analyse the duration of the period between two stoppings of the grain. Considering that the rest periods of the grain are usually the greater parts of the total motion, they can be used as estimates.
The duration of the period can be represented by any real number and, therefore, the phenomenon can be described by a continuous distribution function.

It is also interesting to observe the length of the distance travelled in a grain displacement. This phenomenon can evidently also be described by a continuous distribution function.

As n* can have different values n* = 1, 2,...,k, it is evident that we can analyse the displacement lengths and the periods by the continuous distribution functions. There is a mutual relationship between the families of such functions, because here we have a single random process. Such families and their parameters define also the position distribution function for the grains at a given instant (longitudinal dispersion of the group of the grains of sediment).

DEFINITION OF THE RANDOM PROCESS

Let us follow the motion of the bed-load sediment in a straight channel by observing marked grains immersed in the channel bed at the time $t = 0$. It is not possible to predict with certainty the distance covered by a grain up to the time t.

Therefore, we can write

$$X_t = X_t(t, \omega)$$

as representing a random process for the motion of the sediment. The probability space is an ordered triplet:

$$\{\Omega, \mathcal{A}, P\}$$

where $\Omega = (\omega)$ is the space of the elementary events ω, $\mathcal{A}$ is a σ - algebra the elements of which are certain subsets of the Ω space and P, a probability defined on the family $\mathcal{A}$.

A random trajectory for this process is represented by some functions which characterize the motion of the grain of sediment. The motion of a grain can be represented by a tachograph (Fig.1) or a hodograph (Fig.2).

Such diagrams, characteristic of the motion of sediment, present a succession of periods of rest and displacement. The rest periods are represented on the tachograph by time intervals with a null instantaneous velocity; on the hodographs they are seen as straight segments parallel to the time axis.

We have supposed that grains move only upstream. This means that the trajectories of the random process are non-decreasing monotone functions continuous in time.

In the mathematical sense a trajectory represents an elementary event ω and the space of all possible trajectories is the Ω space. All possible subsets of the Ω space form a σ-algebra. In what follows, we shall analyse certain interesting events for measurable subsets which constitute a σ-algebra, namely:

- the subset of the trajectories of the space Ω, for which the total distance covered up to the instant t is less or equal to x,
- the subset of the trajectories satisfying the condition that the total number of stoppings of the grain during this time interval is less than ν,
- the subset of the trajectories for which the total number of stoppings of the grain in the length interval considered is less than n,
- the subset of the trajectories for which the period for ν cycles is less than t,
- the subset of the trajectories for which the length of the total displacement during n displacement is less than x.

The subsets enumerated define certain important phenomena in the motion of sediment. To describe these phenomena it is necessary to evaluate the probabilities of the indicated subsets, namely, five families of distribution functions.

GRAIN STOPPINGS IN THE TIME INTERVAL

It is not possible to predict with certainty the number of grain stoppings within the time interval (0,t). This phenomenon is considered for the set

$$E_\nu^t = \left\{ \eta_t = \nu \right\}$$

where η_t represents the random number of stoppings from the instant $t = 0$ to t $(t > 0)$.

The characteristics of the set E_ν^t are

$$\forall\, t>0 \qquad E_i^t \cap E_j^t = 0\ , \qquad i \neq j$$

$$\bigcup_{\nu=0} E_\nu^t = \Omega_\eta$$

The phenomenon of stopping time is a marcovian process with following properties:

$$P(E_1^{\Delta t} | E_\nu^t) = \lambda(t,\nu)\,\Delta t + \sigma(\Delta t)\ , \quad \Delta t \downarrow 0$$

$$P(E_0^{\Delta t} | E_\nu^t) = 1 - \lambda(t,\nu)\,\Delta t + \sigma(\Delta t)\ , \quad \Delta t \downarrow 0$$

$$P(E_i^{\Delta t} | E_\nu^t) = \sigma(\Delta t)\ , \quad i \geqslant 2$$

$$P(E_0^0) = 1$$

The probability of ν stoppings of the grain in the interval $(0,t + \Delta t)$ is equal to the sum of probabilities of ν stoppings at $(0,t)$ and zero stoppings at $(t,t + \Delta t)$; the probabilities of $\nu - 1$ stoppings at $(0,t)$ and one stopping at $(t,t + \Delta t)$, etc.:

$$P(E_\nu^{t+\Delta t}) = \sum_{r=0}^{\nu} P(E_{\nu-r}^t\ E_r^{\Delta t})$$

From this equation one can develop partial differential equations by using the given properties of the marcovian process, as follows:

$$\frac{\partial P(E_\nu^t)}{\partial t} = \lambda(t,\nu-1)\,P(E_{\nu-1}^t) - \lambda(t,\nu)\,P(E_\nu^t) \qquad \nu = 1, 2, \ldots$$

$$\frac{\partial P(E_0^t)}{\partial t} = -\lambda(t,0)\,P(E_0^t)$$

The initial conditions are:

$$t = 0 \qquad P(E_0^t) = 1$$

$$t = 0 \qquad P(E_\nu^t) = 0 \qquad \nu = 1, 2, \ldots$$

The solution of differential equations must give the law of the probability of the number of stoppings for a time interval. This solution depends on the form of the stopping intensity function in the interval $\lambda(t,\nu)$, which represents the limiting value of the conditional probability of a grain stopping in a very short interval $(t, t + \Delta t)$, if a grain is stopped several times in the interval $(0,t)$.

In forming the intensity function the nature of the motion should be taken into account. The following postulates must be satisfied:

a) $$\forall t > 0 \qquad \lambda(t,\nu) > 0$$

Which means that during the time t there always is a certain probability for the stopping of the grain in the interval $(t, t + \Delta t)$ if it is stopped ν times in the interval $(0,t)$. This postulate influences the average number of stoppings of the grain in the interval $(0,t)$, defined as:

$$\Lambda(t) = M_{\eta_t} = \sum_{\nu=1}^{\infty} \nu P(E_\nu^t)$$

It can be proved that this is also expressed by

$$\forall t > 0 \qquad \frac{\partial \Lambda(t)}{\partial t} > 0$$

meaning that the increase of the time interval results in the increase of the average number of stoppings of the grain.

b) $$\lim_{t \to \infty} \frac{\partial \Lambda(t)}{\partial t} = \text{const} \neq \infty$$

The limit of the time derivative of the number of stoppings is finite even for large time intervals.

We have investigated many forms of the time intensity function.

The most general form analysed was

$$\lambda(t,\nu) = \lambda_1(t)\lambda_2(\nu)$$

as a product of a time function $\lambda_1(t)$ and a stoppings number function $\lambda_2(\nu)$.

$\lambda_1(t)$ is a continuous analytic function. The integral of $\lambda_1(t)$ is given by

$$\Lambda_1(t) = \int_0^t \lambda_1(s)\,ds$$

The λ_2 (ν) function can be put into the form

$$\lambda_2(\nu) = 1 + \frac{\theta\nu}{1+\nu}$$

In this case one obtains the following solution for the differ ential equation:

$$P(E_\nu^t) = \frac{1}{\nu!\,\theta^\nu} \quad \frac{e^{-\Lambda_1}}{1+\frac{\nu\theta}{1+\nu}} \prod_{i=0}^{\nu} \left[1+i(\theta+1)\right] \sum_{i=0}^{\nu} (-1)^i \binom{\nu}{i} (i+1)^{\nu-1} e^{-\frac{i\theta\Lambda_1}{i+1}}$$

The limit of this expression for large time intervals is:

$$\lim_{t\to\infty} \frac{\partial\Lambda}{\partial t} = (1+\theta) \lim_{t\to\infty} \lambda_1(t)$$

On Fig. 3 we give the intensity functions, the average number of stoppings per time interval and the probability law for the number of stoppings for the preceding form, as well as for the following $\lambda_1(t)$:

$$\lambda_1(t) = \alpha + \beta e^{-\beta t}$$

STOPPINGS OF THE GRAIN PER LENGTH INTERVAL

The random number of stoppings of the grain along a length μ_x from $x = 0$ up to $x (x > 0)$ is described by the random event:

$$G_n^x = \{\mu_x = n\}$$

The characteristics of the set G_n^x are:

$$G_i^x \cap G_j^x = 0 \ , \quad i \neq j$$

$$\bigcup_{n=0}^{\infty} G_n^x = \Omega_\mu$$

The number of stoppings referred to a length interval is a random marcovian process with the following properties:

$$P(G_1^{\Delta x} | G_n^x) = \varkappa(x,n)\,\Delta x + \sigma(\Delta x) \ , \quad \Delta x \downarrow 0$$

$$P(G_0^{\Delta x} | G_n^x) = 1 - \varkappa(x,n)\,\Delta x + \sigma(\Delta x) \ , \quad \Delta x \downarrow 0$$

$$P(G_i^{\Delta x} | G_n^x) = \sigma(\Delta t) \ , \quad i \geqslant 2 \ , \quad \Delta x \downarrow 0$$

$$P(G_0^0) = 1$$

The estimation of the probability of the set G_n^x is analogous to the derivation of the set E_ν^t. One begins by the expression:

$$P(G_n^{x+\Delta x}) = \sum_{r=0}^{n} P(G_{n-r}^x \, G_r^{\Delta x})$$

and ends up with a system of partial differential equations, to wit:

$$\frac{\partial P(G_n^x)}{\partial x} = \varkappa(x, n-1)\, P(G_{n-1}^x) - \varkappa(x,n)\, P(G_n^x) \qquad n = 1,2,\ldots$$

$$\frac{\partial P(G_0^x)}{\partial x} = -\varkappa(x,0)\, P(G_0^x)$$

The initial conditions are:

$$x = 0 \qquad P(G_0^x) = 1$$

$$x = 0 \qquad P(G_n^x) = 0 \qquad n = 1, 2, \ldots$$

In this system of differential equations the intensity function for the stopping of the grain in a length interval appears: $\varkappa(x,n)$. It represents the limiting value of the conditional probability of a grain stopping in a very short interval $(x, x + \Delta x)$, with the condition that the grain would stop n times in an interval $(0,x)$.

The solution of the differential equations depends on the form of the longitudinal intensity function of stoppings.

In developing the longitudinal intensity function it is necessary to satisfy a postulate conditioned by the nature of the motion of the sediment:

- There is always a certain probability on (0,x) for the grain stopping in an interval $(x, x + \Delta x)$, if it has stopped n times in the interval (0,x).

$$\forall x > 0 \qquad \varkappa(x,n) > 0$$

Or, in other form,

$$\forall x > 0 \qquad \frac{\partial \mathcal{K}(x,n)}{\partial x} > 0$$

where $\mathcal{K}(x)$ represent the arithmetic mean of the number of stoppings in the interval (0,x).

We have analysed a more general form of the longitudinal intensity function, represented by the quotient:

$$\varkappa(x,n) = \frac{\varkappa_1(x)}{\varkappa_2(n)}$$

The function $\varkappa_1(x)$ is analysed as

$$\varkappa_1(x) = a(1 - e^{-bx})$$

While $\varkappa_2(n)$ has the form

$$\varkappa_2(n) = 1 + \frac{n}{\rho}$$

In this case, the solution of the differential equations gives the law of the probability of the number of stoppings of a grain in the length interval, as follows:

$$P(G_n^x) = \frac{\rho^n (1 + \frac{n}{\rho})}{n!} \sum_{i=0}^{n} (-1)^{i+n} \binom{n}{i} \left(1 + \frac{i}{\rho}\right)^{n-1} e^{-\frac{\mathcal{K}_1(x)}{1 + i/\rho}}$$

where

$$\mathcal{K}_1(x) = \int_0^x \varkappa_1(s)\, ds$$

On Fig.4 one has the longitudinal intensity functions, the average number of stoppings and the probability law of the number of stoppings of a grain in a length interval for an experiment in a laboratory channel.

DURATION OF THE PERIOD BETWEEN STOPPINGS

Let τ_ν be a random interval during which the grain accomplishes ν cycles of motion.

We assume this to be a continuous random variable, with a distribution function defined by:

$$F_\nu(t) = P\left\{\tau_\nu \leqslant t\right\}, \quad t \geqslant 0$$

For a mathematical derivation we use the known set E_ν^t with a probability defined by:

$$P(E_i^t) = P\left\{\tau_i \leqslant t < \tau_{i+1}\right\}$$

$$P(E_i^t) = P\left\{\tau_i \leqslant t\right\} - P\left\{\tau_{i+1} \leqslant t\right\}$$

By summing the probabilities from $i = 0$ to $i = \nu-1$ one gets:

$$\sum_{i=0}^{\nu-1} P(E_i^t) = \sum_{i=0}^{\nu-1} P\left\{\tau_i \leqslant t\right\} - \sum_{i=0}^{\nu-1} P\left\{\tau_{i+1} \leqslant t\right\}$$

$$\sum_{i=0}^{\nu-1} P(E_i^t) = P\left\{\tau_0 \leqslant t\right\} - P\left\{\tau_\nu \leqslant t\right\}$$

On assuming $\tau_0 \equiv 0$, one gets $P\left\{\tau_0 \leqslant t\right\} = 1$, and, finally, the expression for the distribution function of the period for ν cycles of motion of the grain:

$$F_\nu(t) = 1 - \sum_{i=0}^{\nu-1} P(E_i^t)$$

The probability density follows easily:

$$f_\nu(t) = \lambda(t, \nu-1)\, P(E_{\nu-1}^t)$$

We see that the distribution of the period depends on the probability of the set E_ν^t and, consequently, on the function of time intensity.

With the form adopted for time intensity function:

$$\lambda(t,\nu) = \lambda_1(t)\left(1 + \frac{\theta\,\nu}{1+\nu}\right)$$

one gets the distribution function of the period of ν motion cycles as follows:

$$F_\nu(t) = 1 - \frac{e^{-\Lambda_1}}{(\nu-1)!\,\theta^{\nu-1}} \prod_{i=0}^{i=\nu-1}\left[1+i(\theta+1)\right] \sum_{i=0}^{i=\nu-1} (-1)^i \binom{\nu-1}{i} \frac{(i+1)^{\nu-1}}{1+i(\theta+1)}\, e^{-\frac{i\theta\Lambda_1}{i+1}}$$

LENGTH OF DISPLACEMENT

Let ξ_n be the length of the trajectory described by the grain during n displacements.

Obviously, ξ_n is a continuous random variable, with a distribution function

$$F_n(x) = P\{\xi_n \leqslant x\}$$

The mathematical derivation is analogous to the preceding one, by using the set G_n^x and the expression

$$P(G_n^x) = P\{\xi_n \leqslant x\} - P\{\xi_{n+1} \leqslant x\}$$

One obtains finally the expressions for the distribution function of the trajectory for n displacements of the grain:

$$F_n(x) = 1 - \sum_{i=0}^{n-1} P(G_i^x)$$

and the probability density:

$$f_n(x) = \varkappa(x, n-1)\, P(G_{n-1}^x)$$

It is evident that the longitudinal intensity function plays an important role in the displacement length distribution. We shall present the solution for the distribution functions for displacement lengths for a more general form of the longitudinal intensity function, to wit :

$$\varkappa(x,n) = \frac{\varkappa_1(x)}{1+\frac{n}{\rho}}$$

Here $\varkappa_1(x)$ is a continuous analytic function. Our analysis has borne on a particular form of this function :

$$\varkappa_1(x) = a(1-e^{-bx})$$

where a and b are positive real constants.

For the above form of the longitudinal intensity function one gets the probability density for the length covered by a grain during n displacements, as follows:

$$f_n(x) = \varkappa_1(x)\frac{\rho^{n-1}}{(n-1)!}\sum_{i=0}^{n-1}(-1)^{i+n-1}\binom{n-1}{i}\left(1+\frac{i}{\rho}\right)^{n-2} e^{-\frac{K_1(x)}{1+i/\rho}}$$

POSITION OF THE GRAINS AT A GIVEN INSTANT

Todorović (1966) has shown that the distribution function of the random process in question can be expressed by two approximate functions $F_{t_1}(x)$ and $F_{t_2}(x)$, with the following characteristics:

$$0 \leqslant F_{t_1}(x) \leqslant F_t(x) \leqslant F_{t_2}(x) \leqslant 1$$

The approximations $F_{t_1}(x)$ and $F_{t_2}(x)$ in their most general form are defined by the expressions:

$$F_{t_1}(x) = \sum_{\nu=0}^{\infty}\sum_{j=\nu+1}^{\infty} P(E_\nu^t G_j^x)$$

$$F_{t_2}(x) = \sum_{\nu=0}^{\infty}\sum_{j=\nu}^{\infty} P(E_\nu^t G_j^x)$$

On using the postulate of the independence of the probability of the number of stoppings of the grain during the time interval with relation to the same probability in the length interval :

$$P(E_\nu^t \cap G_n^x) = P(E_\nu^t)\, P(G_n^x)$$

one gets

$$F_{t1}(x) = P(E_0^t) + \sum_{\nu=1}^{\infty} \sum_{j=\nu+1}^{\infty} P(E_\nu^t)\, P(G_j^x)$$

$$F_{t2}(x) = P(E_0^t) + \sum_{\nu=1}^{\infty} \sum_{j=\nu}^{\infty} P(E_\nu^t)\, P(G_j^x)$$

This is a general forms from which the distribution functions of a process X_t can be evaluated as functions of the form of longitudinal and temporal intensity functions.

The mathematical expectation of the position of the grains at a given instant is equal to

$$MX_t = M\xi_1 . M\eta_t$$

with a variance

$$\mathrm{Var}\, X_t = \mathrm{Var}\xi_1 . M\eta_t + (M\xi_1)^2\ \mathrm{Var}\ \eta_t$$

where $M\xi_1$ and Var ξ_1 are respectively the mean and the variance of the distance traversed by a grain during a displacement;

$M\eta_t$ and Var η_t are the mean and the variance of the number of stoppings of the grain during the time interval.

EXPERIMENTS IN A LABORATORY CHANNEL

The tests for our theory were performed in a laboratory channel in Porto Alegre, Brasil, by a team of the Radioactive Researches Institute of Belo Horizonte, during September and December of

1971 and August, 1973. Two kinds of experiments were done:

- a series of tests based on the observation of displacement characteristics of single grains.
- a series based on the registration of the displacement of the group of radioactive grains an the bottom of the channel.

The experiments were performed in a rectangular channel 30 meters long, with a width of 0,40m and the depth of 0,50m; the fixed inclination was 2‰. The channel was fed by water pumped from a large stabilization reservoir with a constant level. The water flow in the channel is measured by a rectangular weir at the entry. At the upstream portion of the channel there is a regulating sluice which maintains the water level.

Before starting the experiments, a level bed 10cm thick was prepared with sand. The upstream and downstream parts were protected over the length of one meter with boards having sand grains glued on and placed on the bottom.

Before the tracing experiment the channel was put in service for several hours in order to establish an equilibrium state for the motion of sediment. In order to maintain this regime after the tracer experiment was started, the sediment was fed in by continuous automatic feeding by two electrical vibrators.

The bed-load sediment transported out of the channel during the experiments was recovered in the sedimentation basin at the downstream end of the channel. In this manner, the flow of sediment was measured precisely.

The material used was natural sand with a semi-angular form, an average diameter of 1,20mm and a parameter of $\sigma \approx 1,26$ for the normal-log law.

During the experiments dunes are formed on the bottom of the channel. Such dunes were registered automatically by micro echosounding. They were observed along the whole length of the channel (throughout the longitudinal profile of the bed), as well as time variations of the profile on a fixed segment of the channel.

During the essays with displacement detection of single grains as tracers, glass grains were used, with a density of 2,67g/cm^3, irradiated until the high specific activity of 20-30μCi by Iridium - 192.

The detection is done by scintillation probes (SPP-3) with a Moseley type recorder with two channels. The scintillation probe was put on wheels and positioned directly over the observed grain during the experiment. The use of a lead collimator permitted a precise measurement of the position (1 to 2cm). The moment of displacement of the grain was determined precisely by the recorder reading. The new position of the grain is determined again 15 seconds later at the most, the detection system being moved manually.

In the series of experiments with radioactive grains, natural sand was used as a tracer marked by Au-198 on the surface of the grains (Courtois-Caillot process).

The tracer is immersed in the bed of the channel in the form of a transverse band 0,5cm thick. We have used 30g of sand marked with a total activity of 5mCi of Au-198.

The detection is done by scintillation detectors mounted on an electric carriage moving along the channel with a speed of 6m/min. The digital radioactivity counter prints the total activity for each 25cm of the channel length.

EXPERIMENTAL RESULTS

Several experiments were performed on single grains under different hydraulic conditions. During a test under the same hydraulic conditions, grains with different diameters were observed.

Based on the experiments and experimental distribution functions, we have analysed the form of time and longitudinal intensity functions, i.e. both the theoretical and experimental parts of the investigation were developed simultaneously.

In order to present the experimental results and the adjustment of experimental results to the distribution functions we give as an example the recordings of the grain D: 307 displacement records in a experiment done in August 1973, with a flow of 25 litres/second.

Fig.5 shows the probability laws for the number of stoppings of the grain during the time interval, while Fig.6 shows the distribution functions of the period for 1, 2, 3 and 4 cycles. Both these theoretical families of distribution functions are defined by the 3 parameters α, β and θ.

Fig.7 presents the probability laws for the number of stoppings of the grain in the length interval x = 20, 30,....180cm. Fig.8 gives the distribution functions of the distances covered during 1, 2, 3 and 4 displacements of the grain. These two families of distribution functions are defined by three new parameters a, b and ρ.

The results of all the tests, for a dune-formed bed, can be summarized as follows:

i - on analysing the number of stoppings of the grain in a time interval, the ratio between the variance and the arithmetic mean is greater than unity,

ii - the increase of an average number of stoppings of a grain in a time interval is not constant:

$$\frac{d\Lambda(t)}{dt} \neq \text{const}$$

iii - the ratio between the variance and the arithmetic mean of the number of stoppings in a length interval is less than unity,

iv - the coefficient of variation of the period during a cycle of motion of the grain is greater than unity.

v - the coefficient of variation of the length during a displacement of the grain is less than unity.

These characteristics of the experimental results have dictated the application of a mathematical model with temporal and longitudinal intensity functions which depend, besides the length of the interval (of time or length, respectively), also on the number of stoppings of the grain in the given interval.

The results of the position detection tests for a group of grains have confirmed the application of the model described above.

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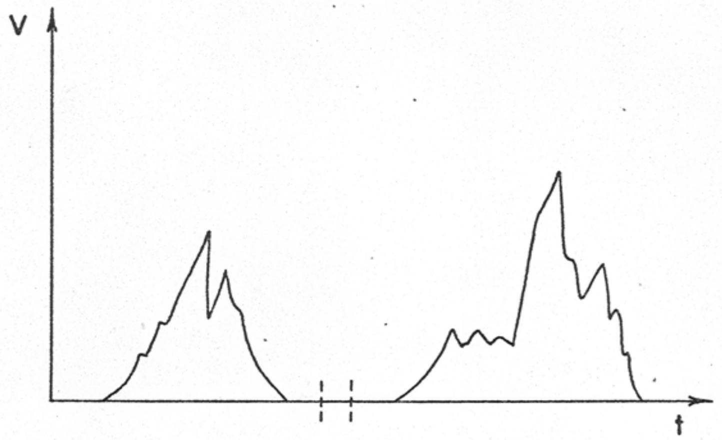


FIGURE 1

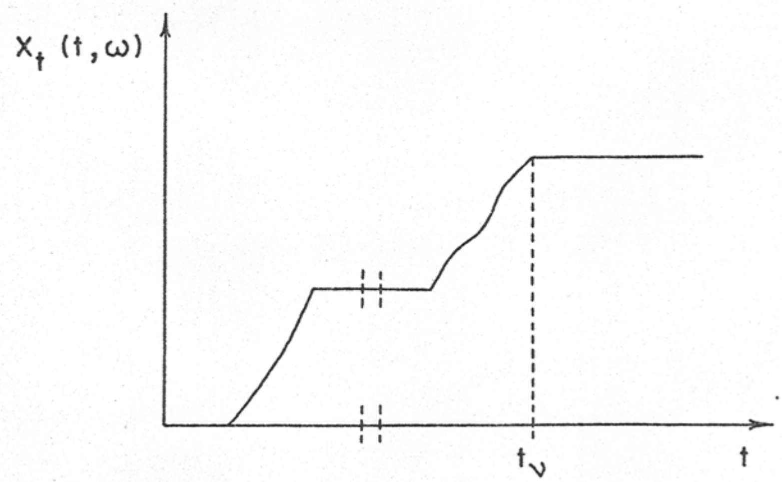
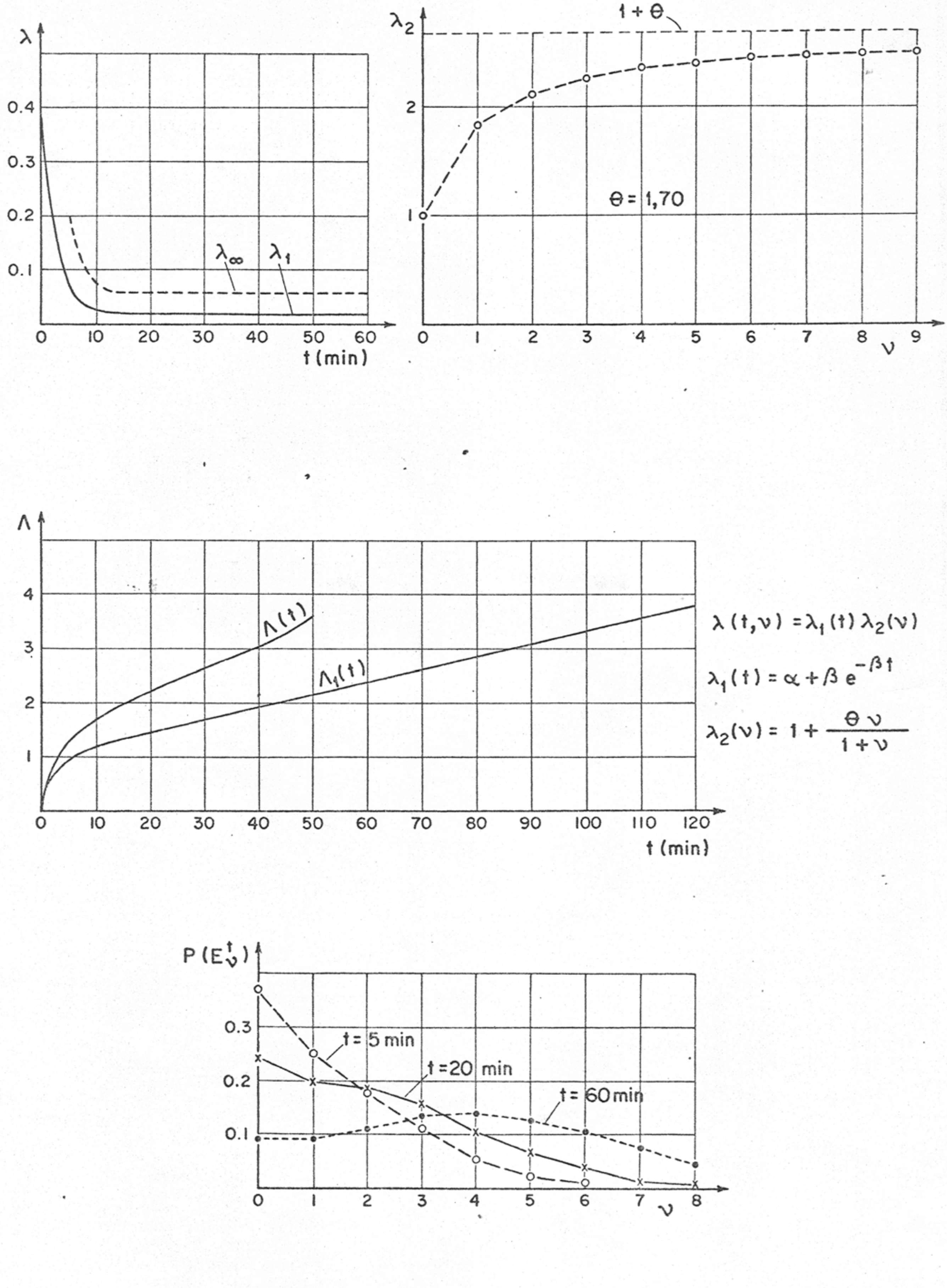


FIGURE 2

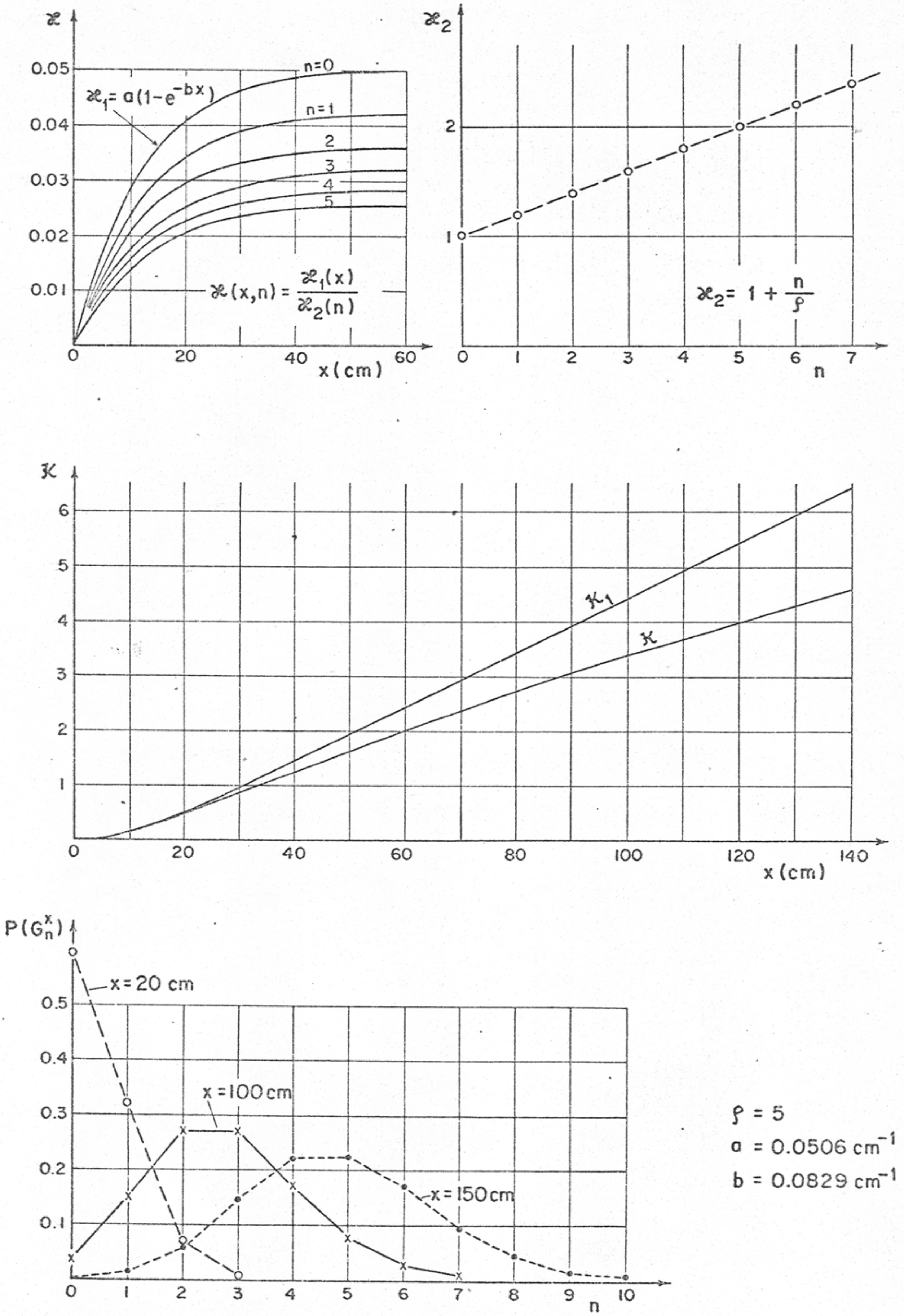


$$\lambda(t,\nu) = \lambda_1(t)\,\lambda_2(\nu)$$

$$\lambda_1(t) = \alpha + \beta\, e^{-\beta t}$$

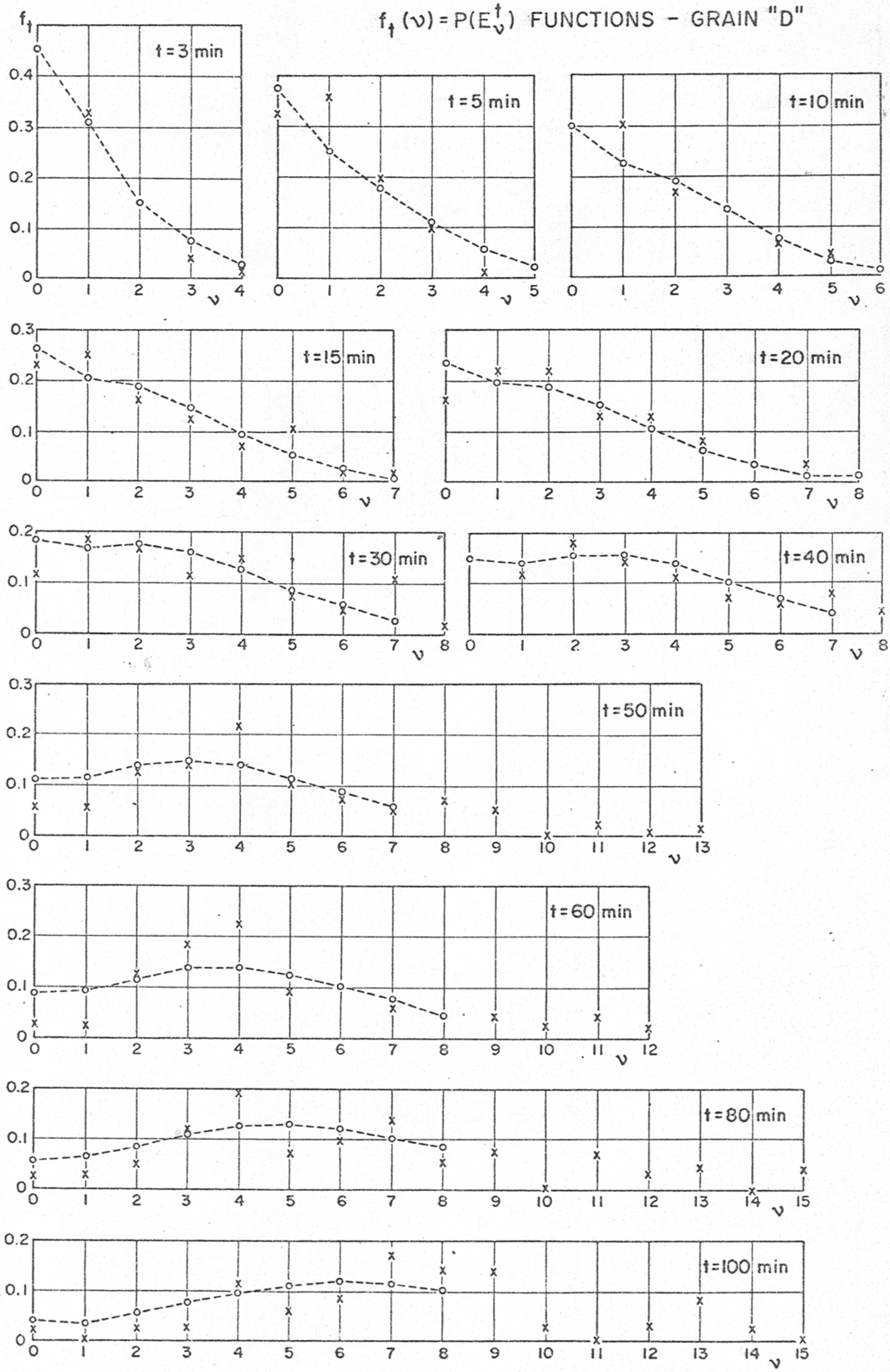
$$\lambda_2(\nu) = 1 + \frac{\theta\,\nu}{1+\nu}$$

FIGURE 3



$\varkappa$
0.05
0.04
0.03
0.02
0.01
n=0
n=1
2
3
4
5
$\varkappa_1 = a(1-e^{-bx})$
$\varkappa(x,n) = \frac{\varkappa_1(x)}{\varkappa_2(n)}$
0
20
40
60
x (cm)
$\varkappa_2$
2
1
$\varkappa_2 = 1 + \frac{n}{\rho}$
0 1 2 3 4 5 6 7
n
$\mathcal{K}$
6
5
4
3
2
1
$\mathcal{K}_1$
$\mathcal{K}$
0
20
40
60
80
100
120
140
x (cm)
$P(G_n^x)$
0.5
0.4
0.3
0.2
0.1
x = 20 cm
x = 100 cm
x = 150 cm
0 1 2 3 4 5 6 7 8 9 10
n
$\rho = 5$
$a = 0.0506\ cm^{-1}$
$b = 0.0829\ cm^{-1}$

FIGURE 4



$f_t(\nu) = P(E_\nu^t)$ FUNCTIONS - GRAIN "D"
f_t
t=3 min
t=5 min
t=10 min
t=15 min
t=20 min
t=30 min
t=40 min
t=50 min
t=60 min
t=80 min
t=100 min
ν

FIGURE 5

$F_\nu(\tau)$ FUNCTIONS - GRAIN "D"

$\alpha = 0.0231 \text{ min}^{-1}$

$\beta = 0.4200 \text{ min}^{-1}$

$\theta = 1.7$

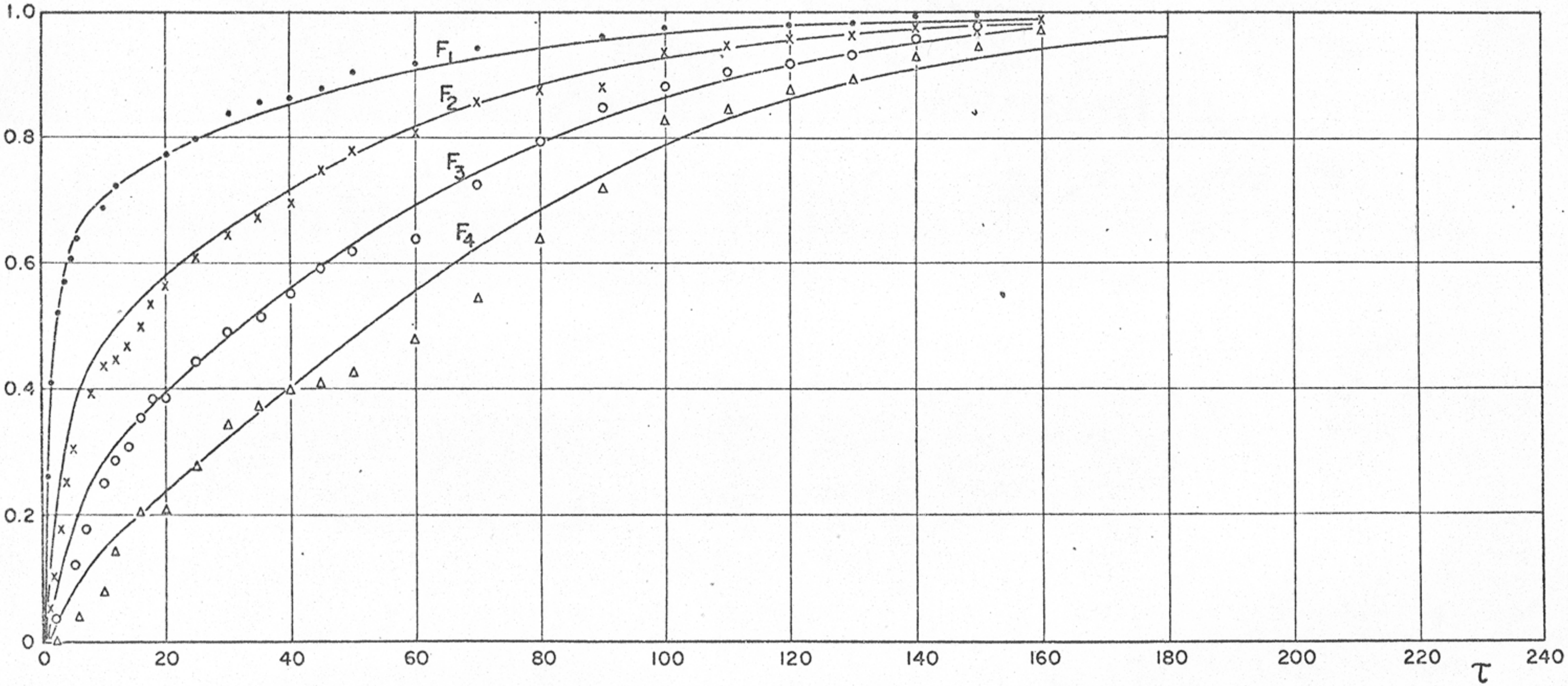


FIGURE 6

$f_x(n) = P(G_n^x)$ FUNCTIONS — GRAIN "D"

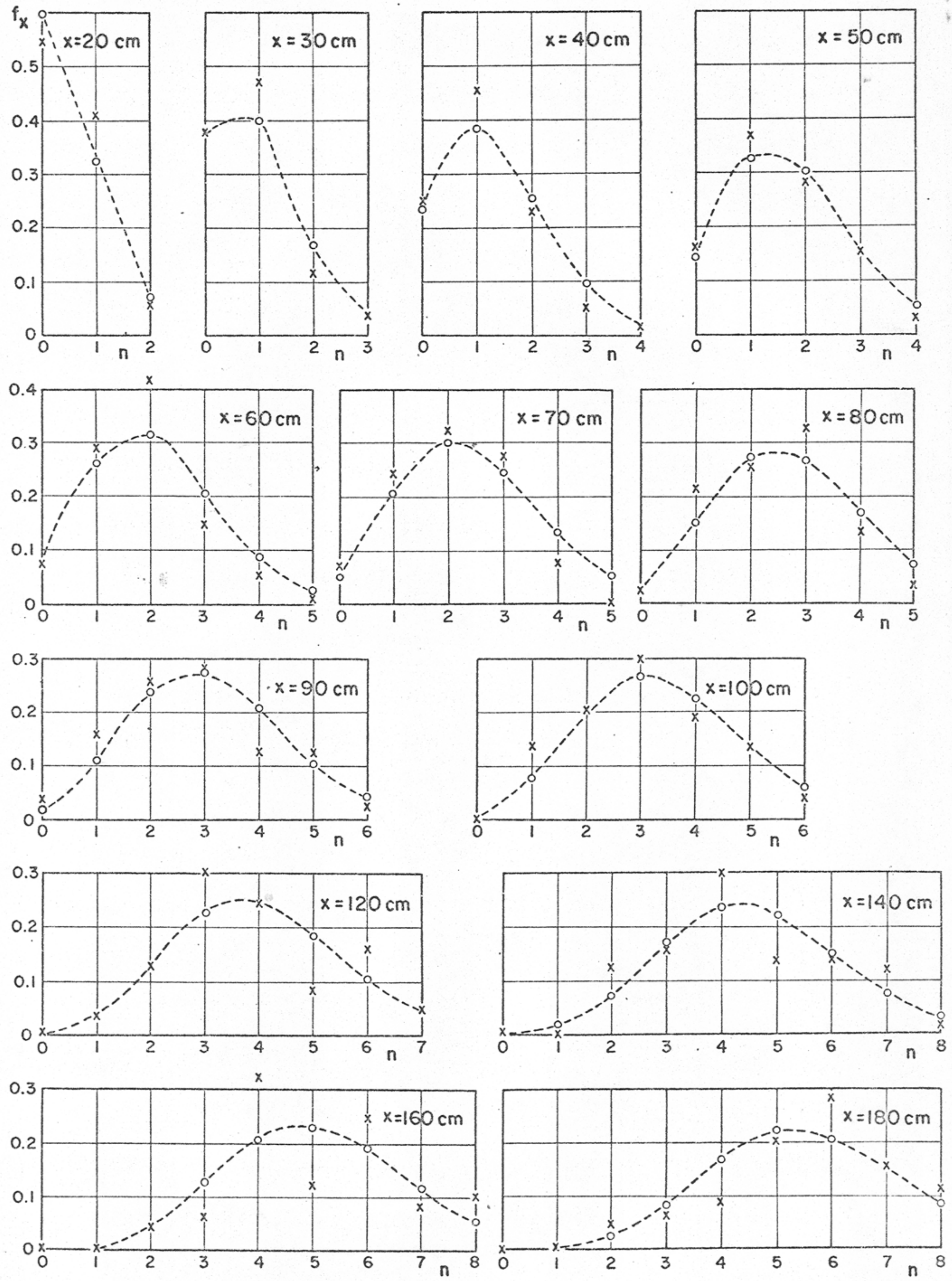


FIGURE 7

$F_n(\xi)$ FUNCTIONS - GRAIN "D"

$a = 0.0506\ cm^{-1}$

$b = 0.0829\ cm^{-1}$

$\rho = 5.0$

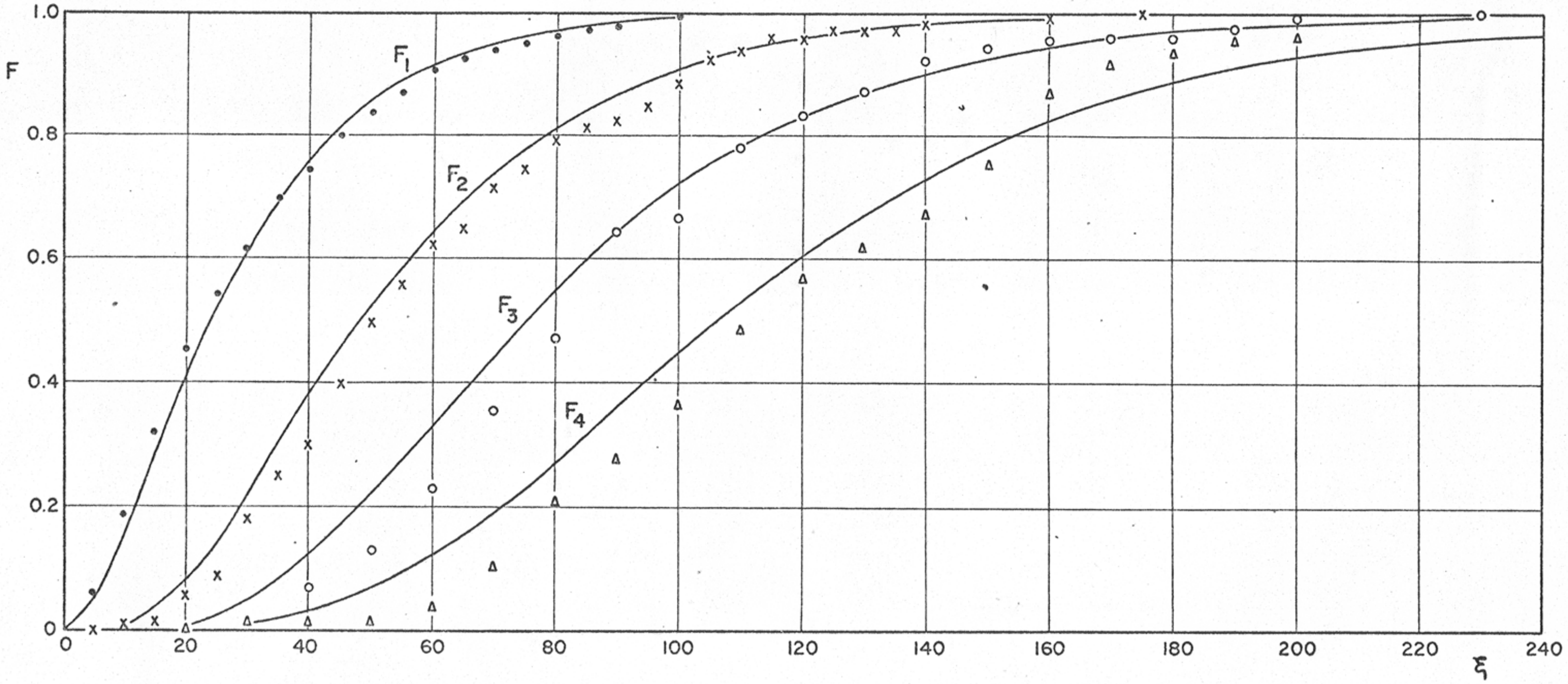


FIGURE 8