Progress in Nuclear Energy 99 (2017) 119–126

Contents lists available at ScienceDirect

Progress in Nuclear Energy

journal homepage: www.elsevier.com/locate/pnucene

Support to the identification of anomalies in an external neutron source using Hurst Exponents

Edson Henrice Jr. [a,*], Daniel A.P. Palma [b], Alessandro C. Gonçalves [a], Amir Z. Mesquita [c]

[a] PEN/COPPE, Av. Horácio Macedo, 2030, Rio de Janeiro, RJ, 21941-941, Brazil
[b] CNEN, Rua Gal Severiano, 90, Rio de Janeiro, RJ, 22290-901, Brazil
[c] CDTN, Av.Pres. Antônio Carlos, 6.627, Belo Horizonte, MG, 30161-970, Brazil

ARTICLE INFO

Article history:
Received 23 June 2016
Received in revised form
4 April 2017
Accepted 15 May 2017
Available online 24 May 2017

Keywords:
Neutron flux
Fractional diffusion
Fractional derivative
Spurious Beam
Hurst exponent
Detrendend Fluctuation Analysis
Rescaled Range

ABSTRACT

A new methodology is proposed here to identify anomalies in the neutron flux as a result neutron production trips in an Accelerator Driven System - ADS type reactor. This methodology is based on the calculation of Hurst exponents, where the neutron flux as monitored in the reactor core is treated as a temporal series. In several recent articles, related to fractional diffusion, the Hurst exponent is used as an estimate for the order of the fractional derivative. Our object of study considered a reactor based on the Myrrha simulated with the Serpent Monte Carlo code for two kinds of trips occurred in the production of neutrons, as follows: a Peak of Production (PP), the Unprotected* Accelerator Beam Overpower (UABO), and the spurious Beam Trip (BT). In order to estimate the Hurst exponent it was used two different methods, namely the Rescaled Range Analysis (R/S) and the Detrended Fluctuation Analysis Method (DFA). The results obtained showed that the R/S methodology had some advantages in a comparison with the DFA in indicating the occurrence of those anomalies. Also being able to provide a scale for the assessment of the intensity of the trip occurred, showing to be an useful tool to support anomaly identification in the neutron flux of ADS reactors.

1. Introduction

Amongst 4th-gen reactor designs the one the stands out for its capability to generate power from transmuting heavy elements with a long half-life, and thus reducing the inventory of radioactive materials, is the Accelerator-Driven System (ADS). ADS reactors have a sub-critical core guided by a beam high-energy protons generated by a linear accelerator. These protons, as they collide with an appropriate target, generate, from spallation reactions, the neutrons needed to keep the fission rate of the cores under control, to obtain power (Mukaiyama et al., 2001). Following the spallation reaction the neutrons are diffused in the reactor core, providing a power production dynamic that is being widely studied as this proposal will still be used in a large scale.

One of the goals of this paper is to identify, through the neutron flux, the instant when anomalies of the UABO and BT kind take place (Suzuki et al., 2005; Vandeplassche et al., 2011). In order to identify an anomaly in the neutron flux a different approach is presented here that allows considering small statistical fluctuations. For that, we will use the time series studies initially done by Hurst et al. (1966). These methods will be used to identify trends in time series and to classify the diffusion processes occurred in this type of system. The evaluation of the Hurst Exponent will be made from two different calculation methods, namely: Rescaled Square (R/S) (Hurst et al., 1966) and Detrendend Fluctuation Analysis (DFA) (Peng et al., 1994).

To that end we simulated a Myrrha Reactor, using the Serpent Reactor Physics code, after Bruyn et al. (2007), considering a neutron source that presents anomalies of the UABO and BT kinds. We resorted to the Serpent code as it allows the implementation of several counters on any surface or in any region of the reactor under study. With it one can get the neutron flux during the operation of the reactor at hand with detail.

Section 2 describes the methodology used to calculate the Hurst Exponents, using the R/S and DFA methods. Section 3 describes the simulations we carried out. Section 4 provides the preliminary results as obtained in the identification of UABO and BT anomalies,

* Corresponding author.
E-mail addresses: edonhenrice@yahoo.com.br (E. Henrice), dapalma@cnen.gov.br (D.A.P. Palma), alessandro@con.ufrj.br (A.C. Gonçalves), amir@cdtn.br (A.Z. Mesquita).

http://dx.doi.org/10.1016/j.pnucene.2017.05.009
0149-1970/

whilst Section 5 provides the conclusions to this work.

2. Methodology

The neutrons have a random trajectory and the stochastic formulation of these transport phenomena, as regards a random path, as well as the description through a mathematical model executed through the diffusion equation are two fundamental concepts in the theory of diffusion. The linear dependency on the temporal growth of the average quadratic displacement of the particles $\langle x^2(t)\rangle \propto tS$ or, in an equivalent manner, of variance, is a characteristic of Brownian motion and therefore of regular diffusion. The characteristic of anomalous diffusion is generally a non-linear growth of the variance in time, that is, $\langle x^2(t)\rangle \propto t^{\alpha}S$. Several works on anomalous diffusion in the field of reactor Physics can be found in the literature (Espinosa-Paredes et al., 2011; Vyawahare and Nataraj, 2013) that discuss possible advantages in relation to conventional diffusion equations based on integer order derivatives, and consider an infinite speed of the neutrons, and have a limited spatial application.

With this, it is necessary to use a tool capable of estimating the value of α. In a recent article, Hahn et al. (2010) proposed that the order of the fractional derivative is twice that of the Hurst exponent, and Espinosa-Paredes et al. (2006) used the DFA methodology directly, to obtain the order of the fractional derivative without going much deeper in it. The original problem as studied by Hurst was linked to the construction of dams in the River Nile and for that he monitored its periods of greater and smaller flow. Hurst wanted to establish the ideal volume for water storage in a reservoir, based on the data for water as required in time.

Let's call the Hurst exponent in a generic way as H. Two methodologies are presented below to estimate this exponent, as used in this article.

2.1. The R/S method

The problem tackled in Hurst's analysis consisted of determining a model for a reservoir, based on the records for output flow for a given lake so that the reservoir neither emptied completely nor overflowed. In a given year t, the lake receives a random influx of water $\xi(t)$ as a result of the rainfall. A mean value for the water flow rate $\langle\xi\rangle_\tau$ has to be released by the reservoir so to maintain a certain volume of water in storage. Therefore, the mean value for the output flow rate in relation to the time frame should be:

$$\langle\xi\rangle_\tau = \frac{1}{\tau}\sum_{t=1}^{\tau}\xi(t), \tag{1}$$

with $X(t,\tau)$ for accumulated discharges, obtained by subtracting the value of the discharge from its mean value, that is:

$$X(t,\tau) = \sum_{u=1}^{t}\{\xi(u) - \langle\xi\rangle_\tau\}(1 \le t \le T). \tag{2}$$

The difference between the maximum and minimum discharge numbers, X, is the average for the R (range).

$$R(\tau) = \max_{1\le t\le\tau}X(t,\tau) - \min_{1\le t\le\tau}X(t,\tau), (1 \le t \le \tau), \tag{3}$$

where $R(\tau)$ is the variation between the maximum and the minimum volumes of the reservoir, and τ is the time frame. Thus, R is the storage capacity needed to keep the minimum discharge volume in the period so to meet the requirement that the reservoir neither overflows nor dries out completely.

Hurst studied many natural phenomena, using the a-dimensional ratio R/S where S is the standard deviation, that is, the root square of the variation, as provided by:

$$S = \left(\frac{1}{\tau}\sum_{t=1}^{\tau}\{\xi(t) - \langle\xi\rangle_\tau\}^2\right)^{\frac{1}{2}}. \tag{4}$$

Hurst found that the a-dimensional ratio R/S allows comparing the re-sizing of several temporal series and that such a re-sizing can be very well described by a law of power as follows:

$$\frac{R}{S} = kN^H, \tag{5}$$

where: N is the time interval for the observations, H is the estimate for Hurst's Exponent as calculated from the R/S method and k is a constant.

This method is very well established in the literature and has applications in many areas of knowledge, such as the stock market (Sánchez et al., 2015), hydrology studies (Koutsoyiannis et al., 2011), and on multi-phase flow systems (Li et al., 2013).

2.2. DFA - Detrended Fluctuation Analysis

The determining of Hurst's exponent has been widely studied, giving rise to a large number of methodologies for the analysis of temporal series, including the so-called Detrended Fluctuation Analysis (DFA) (Peng et al., 1994). A brief description of this technique is given below:

i) From the original series one obtains the integrated series;

$$x(i) = \sum_{i=1}^{k}[y_k - \langle y\rangle]. \tag{6}$$

The integrated series is divided into N windows with n elements in each window;

ii) The trend of the series is removed by reducing from the original series the polynomial $p_v(i)$ as adjusted in the window;

$$x_n(i) = x_i - p_v(i). \tag{7}$$

iii) The variance for each segment is calculated;

$$F_n(v) = \langle x_n(i)\rangle = \frac{1}{n}\sum_{i=1}^{n}x_n[(v-1)n + i]. \tag{8}$$

iv) The average is calculated for all the segments and the square root is found to obtain the DFA fluctuation function:

$$F(n) = \sqrt{\frac{1}{N}\sum_{v=1}^{N}(F_n(v))^2}. \tag{9}$$

This calculation is repeated for all possible window sizes, from $n_{max} = N/4$ to $n_{min} = 5$. Usually $F(n)$ increases with the number of n window elements. After that, a graph $\log(F(n))$ x $\log(n)$ is plotted and the linear coefficient of this graph determines the self-similarity parameter ω in the shape of an exponent N^H, where H is Hurst's Exponent, as estimated from the DFA methodology.

For a short interval or for situations where the points are not correlated, one obtains $H = 0.5$ or 'white noise'. On the other hand, if $H < 0.5$ the correlation in the signal is anti-persistent, that is, an increase is probably followed by a decrease and vice-versa. For

$H > 0.5$ the signal correlation is persistent, an increase probably being followed by another increase and vice-versa. In this work the said methodologies were implemented with the use of the Octave open-source software to obtain the H exponent through the two previously described methodologies.

3. Simulations

To estimate the Hurst exponent, calculated respectively with the R/S and DFA methods, we simulated an ADS reactor based on the geometry and structure described in Bruyn et al. (2007). In Fig. 1, obtained from simulations with the Serpent code, it is possible to see a cross-section of the Myrrha based reactor.

The reactor has 99 fuel elements, each one filled with 91 rods. The fuel rod parameters are shown in Table 1. The composition of the materials used in the simulation is shown in Tables 2–4.

Several simulations were carried out in this work to evaluate the behaviour of the neutronic flux, with the methodologies presented. The Serpent code allows inserting neutron flow counters in any region of the reactor. In the study case we chose the central annular section to insert these counters.

The counters provide the quantity of neutrons that go over a surface in a given time frame. In general, the analyses were done by dividing the total time for the simulated operation of 100 s into 10^5 intervals to implement the counters. Once this model was obtained, it was possible to apply the R/S and DFA methods to estimate the order of Hurst's exponent.

To run the DFA calculation we used a routine found in Weron (2015), and to calculate the R/S method we used an internal routine of the Octave 3.8.2 code. In order to apply the method, the data generated by the counters in the 100-s period of simulation was split into different time windows (bins). Power reactor was kept constant, at 50 MW, in all cases.

4. Results

4.1. Default case – constant source

The first case simulated aims at evaluating the normal operating conditions pursuant to the DFA and R/S methodologies, to be a reference for the other ones. To that end, the external neutron source is considered as constant for a period of 100 s. In a first analysis the studies were done for 10 and 20 bins and in each one of them the methodologies presented were applied, to then produce a graph with 10 or 20 points, representing the 100 s analysed.

Fig. 2 shows the temporal evolution of Hurst's exponent for 100 s split into 10 bins, that is, each point representing the calculation for Hurst's exponent, as done with all the flux counters contained in the 10-s interval. Fig. 3 shows the temporal evolution of Hurst's exponent for 20 bins.

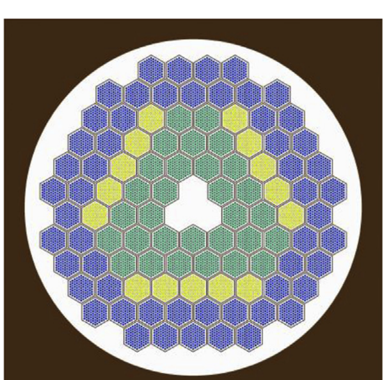

Fig. 1. Horizontal section in simulated reactor.

Table 1
General parameters of the fuel rod.

Parameter	Size (cm)
Radius of fuel	0.3075
Inner radius of the fuel rod	0.3175
Outer radius of the fuel rod	0.3275
Pitch between the fuel rods	0.855
Pitch between the fuel assembles	9.62
Active Height	120

It is possible to see, based on Figs. 2 and 3 that the behaviour of Hurst's exponent as calculated with both methods is similar. The difference that can be more clearly seen is that the R/S method overestimates the results obtained in comparison with the DFA method for the majority of the points calculated. In both the methodologies used to analyse the Hurst exponent it was found that the points are distributed at around 0.5, that is, where the methodology did not find correlated points.

4.2. Anomaly in the neutron source I: Peak of Production (PP)

In this case a 20% neutron production peak is introduced for two seconds, as shown in the histogram in Fig. 4.

For some more subtle neutron production trips the neutron flux rarely displays any detectable variation, thus making it necessary to apply some methodology such as those that have been used in this work to correctly identify these anomalies from the neutron flux.

Figs. 5 and 6 show the temporal evolution for Hurst's exponent with the use of 10 and 20 equal time intervals (bins). It is possible to see, according to the methodology, the occurrence of an anomaly in the neutron flux as a result of a peak in neutron production from the spallation reactions, such as that described in Fig. 4. This simulation considered a $k_{eff} = 0.955$.

Based on the result shown in Fig. 5 one can see that both methods were able to clearly identify the bin in which the anomaly took place. The same cannot be said for the results shown in Fig. 6 where the DFA method had a peak 24.8% lower than that we obtained with the use of the R/S methodology. This means that, in this interval, one cannot say some kind of anomaly took place, in the comparison to the other points, according to the DFA methodology. Further, based on Fig. 5 one can see that the reactor quickly returns to a stable condition of normal operation, which confirms the safety inherent to subcritical reactors guided by an external source. This finding becomes clear in Fig. 6 where a smaller bin was taken into account. Still, for the case of a production-peak type anomaly other k_{eff} values were considered, as obtained by the variation on core composition. The values displayed for the bin where the anomaly is present can be seen in Table 5.

From the percentage deviations shown in Table 5 it is possible to see that the identification of the anomaly in the neutrons source is easier with the R/S method than with the DFA method for any K_{eff} considered. It is possible to see that when the system is farther from a state of criticality, it is more susceptible to instabilities caused by the neutron beam trips.

4.3. Anomaly in the neutron source II: Unprotected Overpower Beam

In the case of the UOB, a sharp increase of 100% in the neutrons production as originated from the spallation reactions was considered, at a 22.5-s instant, remaining stable on this new level,

Table 2
Fuel composition (density = 11 g/cm^3).

Element	Fuel-in (mass fraction)	Fuel-middle (mass fraction)	Fuel-out (mass fraction)
O	0.1185	0.1185	0.1185
U-235	0.0521	0.0021	0.1121
U-238	0.7503	0.7853	0.7143
Pu-238	0.0010	0.0010	0.0010
Pu-239	0.0343	0.0493	0.0353
Pu-240	0.0108	0.0108	0.0108
Pu-241	0.0301	0.0301	0.0051
Pu-242	0.0030	0.0030	0.0030

Table 3
Coolant composition (density = 10 g/cm^3).

Element	Mass fraction
Pb-82	0.45
Bi-83	0.55

Table 4
Reflector composition (density = 5.3908 g/cm^3).

Element	Atomic Density (atoms/barn. cm)
Na-23	7.15E+01
Fe-54	2.77E+01
Fe-56	4.35E+02
Fe-57	1.01E+01
Fe-58	1.34E+00
Ni-58	1.99E+00
Ni-60	7.67E-01
Ni-61	3.33E-02
Ni-62	1.06E-01
Ni-64	2.71E-02
Cr-50	3.06E+00
Cr-52	5.91E+01
Cr-53	6.70E+00
Cr-54	1.67E+00
Mn-55	3.12E+00
Zr	3.33E+00

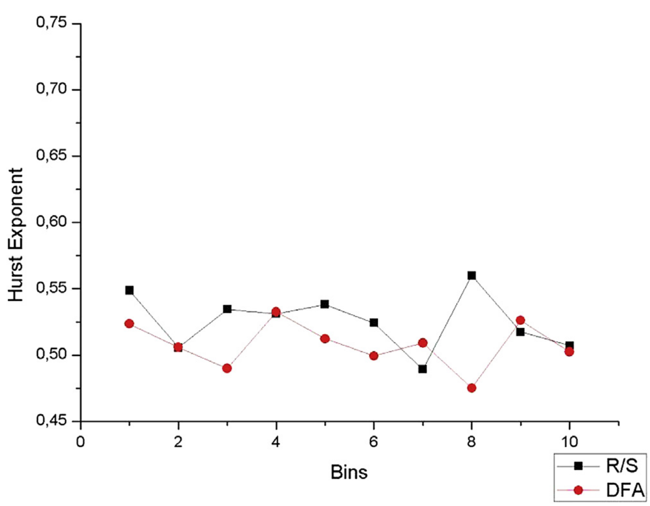

Fig. 2. Temporal evolution of the Hurst exponent calculated from the methodologies R/S and DFA considering 10 bins of 10 s each.

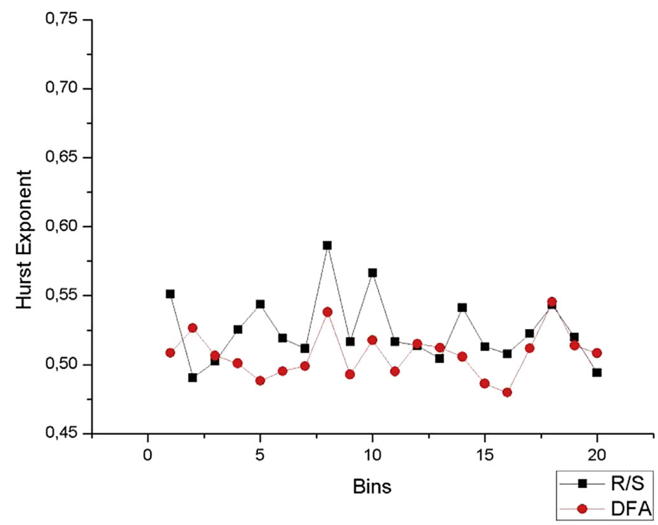

Fig. 3. Temporal evolution of the Hurst exponent calculated from the methodologies R/S and DFA considering 20 bins of 5 s each.

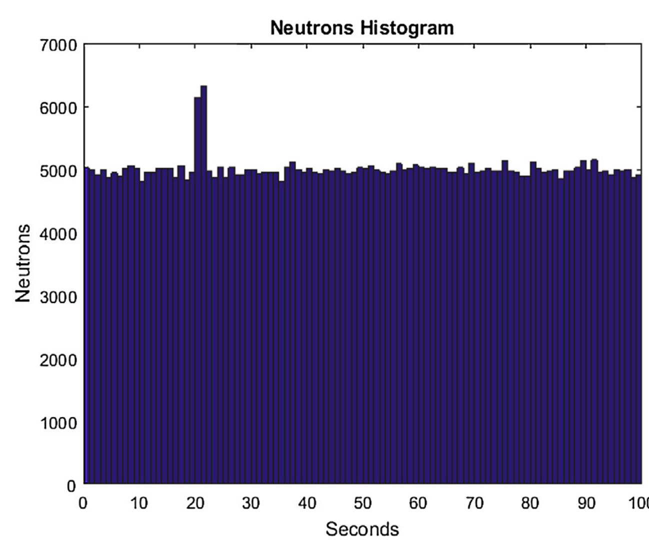

Fig. 4. Histogram of the neutron source with 20% production peak.

as shown in Fig. 7.

The results produced with the DFA and R/S methodologies can be seen in Figs. 8 and 9.

With it, it was possible to easily identify which bin it had taken place in. In order to better observe the relation between the value of the coefficient obtained by the DFA and R/S methods, in the analyses of the neutron flux, and the increase in intensity for neutron generation in the shape of a step, new simulations with variations in the neutron supply other than those shown in Fig. 7 were carried out. The result is shown in Table 6, where it is possible to see that the value for the Hurst Exponent is related to the variation in

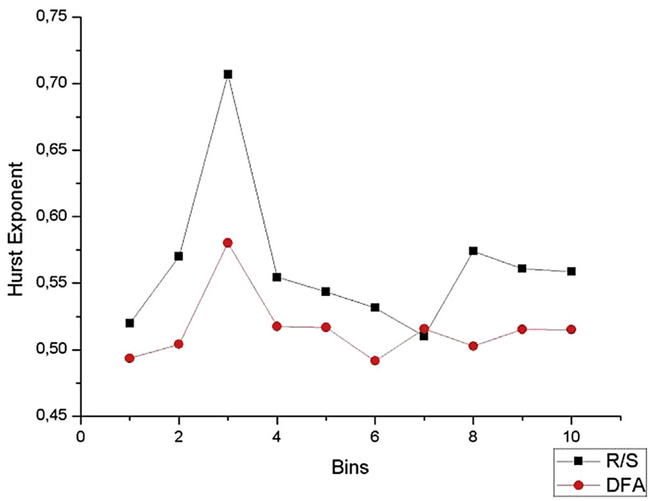

Fig. 5. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 10 bins with 10 s each.

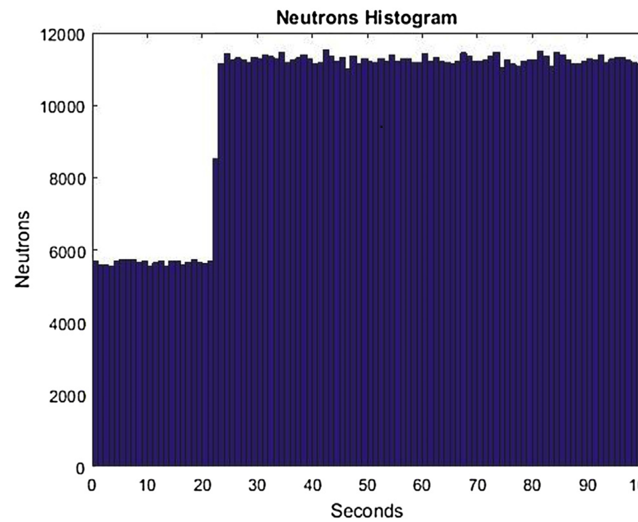

Fig. 7. 100% increase in the neutron production in the instant 22.5s.

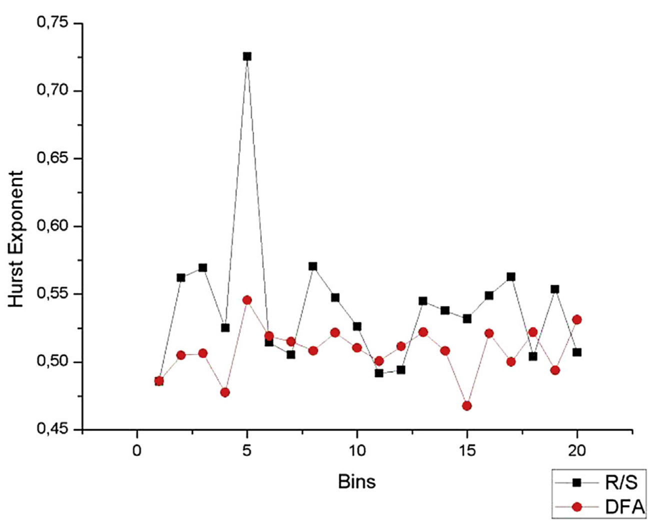

Fig. 6. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 20 bins with 5 s each.

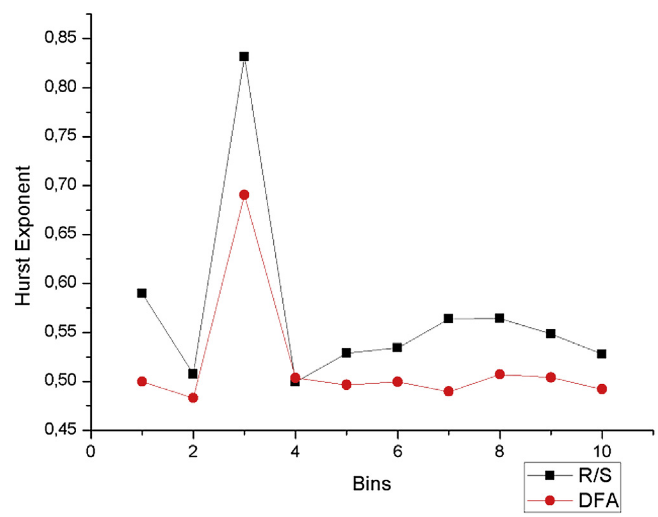

Fig. 8. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 10 bins with increased neutrons production of 100% at the 22.5s instant.

Table 5
Peak value of the Hurst's exponent considering different k_{eff} calculated respectively with the R/S and DFA methods.

k_{eff}	10 Bins			20 Bins		
	MX-RS	MX-DFA	Difference %	MX-RS	MX-DFA	Difference %
0.981	0.69	0.57	17.3	0.70	0.55	28.1
0.973	0.70	0.57	18.6	0.71	0.54	32.6
0.963	0.72	0.60	16.2	0.73	0.57	27.7
0.955	0.72	0.61	15.7	0.75	0.58	28.6

neutron production.

4.4. Anomaly in neutron source III: Spurious Beam Trip

In the case of the SBT, we first simulated a two-second interruption in neutron generation from the twentieth second, as shown in Fig. 10.

With the neutron generation behaving as shown in Fig. 10, it was possible to obtain the results shown in Figs. 11 and 12 with the DFA and R/S methods, where it is possible to detect an anomaly produced in the distribution of the neutron flux in the reactor, as a result of the interruption of the neutron supply.

It is possible to see from Figs. 11 and 12 and in Table 7 that the R/S methodology was more efficient when showing where each anomaly took place. This behaviour repeated itself in the other cases studied and, in the next subsection, where the online anomalous flux identification is dealt with, only the R/S method will be considered.

4.5. Online flux identification

The case that drew the most attention in the study of anomalies in the neutron flux in ADS reactors is that of the identification of an interruption in the supplying of neutrons by an external source. With it, the SBT case is probably the most interesting to be studied as, according to the bases of the project, the incident where the

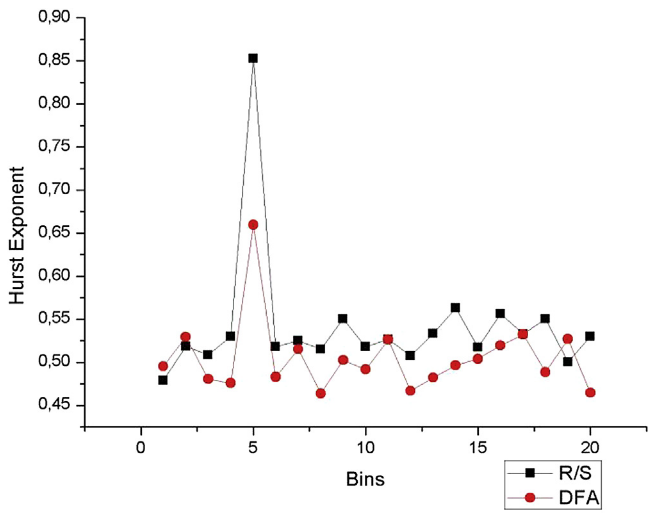

Fig. 9. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 20 bins with increased neutrons production of 100% at the instant 22.5s.

Table 6
Coefficient of R/S and DFA obtained during the rise in neutron production as a step.

Increasing on neutrons production in %	10 bins		20 bins	
	R/S	DFA	R/S	DFA
100	0.84129	0.69043	0.85299	0.65941
75	0.83195	0.69841	0.85215	0.65350
50	0.79077	0.6433	0.80767	0.61691
30	0.74376	0.60095	0.75914	0.57603
10	0.64210	0.55121	0.64496	0.55883

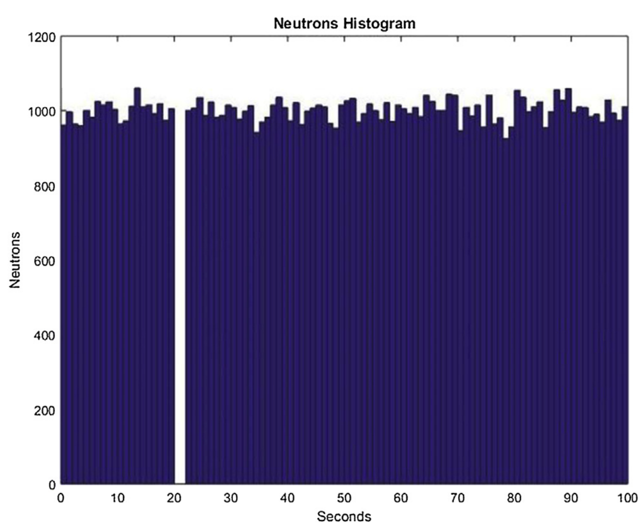

Fig. 10. Interruption in neutron generation during 2s.

neutron source is non-operational for more than 3 s should not occur more than 10 times during an operating cycle.

In order to carry these studies out, a simulation was made of an interruption in the neutron supply for just 0.1 s in the 19.9 to 20 s-interval. This way the interruption always occurs at the end of the bin that is being analysed. With it, we expect to see the continuous monitoring of the neutron flux with a time frame history that is relatively short, where the data can be acquired in a continuous way and where the flux is detected in 0.1 s, thus allowing the online identification of interruption in the external source by just observing the neutron flux.

In evaluating the methodology chosen, and as regards the changes in data acquisition, the studies in this section were also done for 10^4 counters in the 100-s period.

As it can be seen in Figs. 13 and 14, with a 0.1-s interruption in the neutron supply at the end of the bin under analysis, the methodology is able to identify anomalies in the system, through a comparison with the remaining points obtained. As regards the time interval for data acquisition, the methodology presented was efficient as it identified the occurrence of the anomaly for 20 and 100 bins in a 100 s simulated time interval.

5. Conclusions

A new methodology to identify anomalies in the external neutron source in ADS reactors with the use of in-core detectors is presented in this work. In a general way, the neutron flux in the core is available in all reactors, this being a basic bit of information, generally monitored online. The method presented in this paper uses this already-available data and, after mathematical treatment, identifies an anomalous behaviour, caused by the external neutron source. To this end, two different analysis methods were used, based on the calculation of the Hurst Exponent, namely: Rescaled Range Analysis (R/S) and the Detrended Fluctuation Analysis (DFA) methods.

In order to obtain the data needed to calculate the Hurst exponent a subcritical system was simulated with the Serpent code, based on the Myrrha Reactor. This system was subjected to different kinds of trips in the neutron source: Peak of Production (PP), Unprotected Accelerator Beam Overpower (UABO), and Spurious Beam Trip (SBT).

The results obtained showed that the two methodologies described in this work could be used as a tool to support the identification of anomalies in the neutron flux, also being able to providing an estimate of the intensity of the trip occurred and, with it, being valuable tools to support the identification of such anomalies.

However, and based on the data presented in this work it is

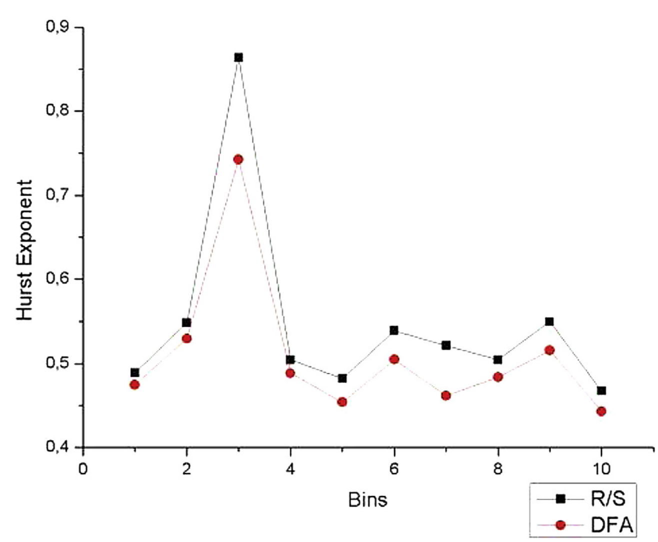

Fig. 11. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 10 bins and an interruption in the neutrons production for 2 s occurring after the twentieth second.

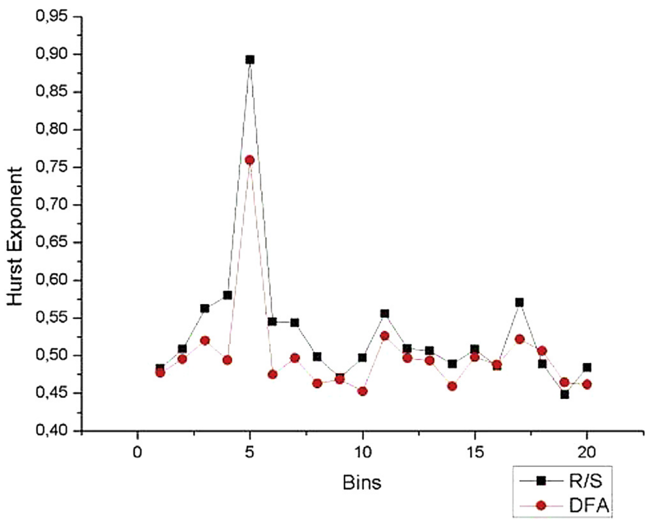

Fig. 12. Temporal evolution for Hurst's exponent calculated from the R/S and DFA methodologies considering 20 bins and an interruption in the neutrons production for 2 s occurring after the twentieth second.

Table 7
Peak value for the coefficient of R/S and DFA obtained during the interruption of the neutron supply.

Interruption source time (s)	Peak value using 10 bins		Peak value using 20 bins	
	R/S	DFA	R/S	DFA
1	0.81124	0.77919	0.85225	0.77106
2	0.86409	0.74257	0.89293	0.75946
3	0.88854	0.80341	0.89994	0.82801

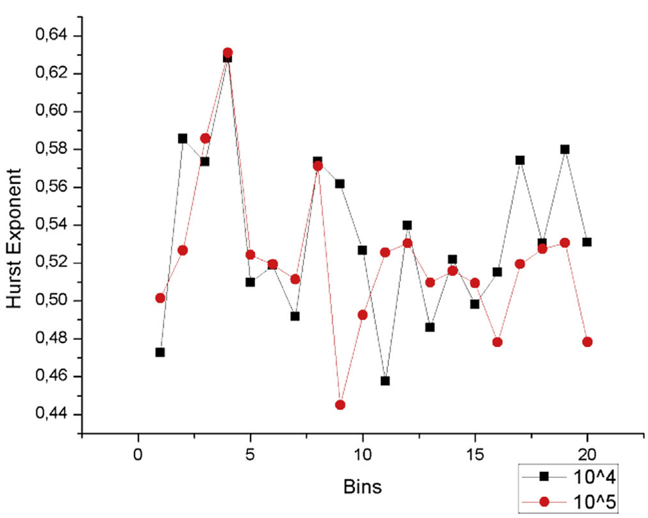

Fig. 13. R/S Method applied to 20 bins with data obtained by 10^4 and 10^5 counters in a time interval of 100 s and 0.1 s interruption at the end of the fourth bin.

possible to see that the results obtained in the calculation of the Hurst Exponents with the R/S method systematically overestimates those obtained with the DFA methodology. Moreover, it was show to be more efficient in providing an estimate of the level of disturbance caused to the neutron flux in the reactor, taking into account factors such as the k_{eff} and the intensity of the trip as

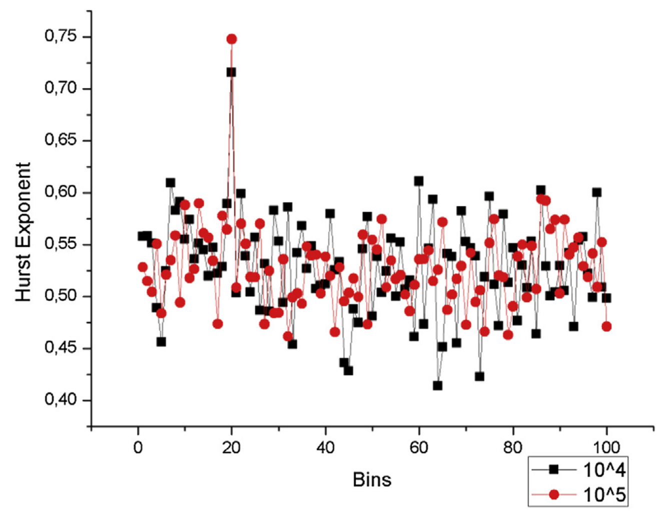

Fig. 14. R/S Method applied to 100 bins with data obtained by 10^4 and 10^5 counters in a time interval of 100 s and 0.1 s interruption at the end of the nineteenth bin.

caused in the neutron source. As regards online detection, the R/S methodology had good results in identifying the occurrence of anomalies in the neutron flux, even when using a short time interval after their happening.

As a result, the authors recommend the use of the R/S method to identify changes to the neutron flux as a result of anomalies in the external neutron source, in ADS reactors. This methodology could also be applied to identify other events that affect the neutron flux in the reactor.

Acknowledgement

This research project is supported by the following Brazilian institutions: Research Support Foundation of the State of Minas Gerais (FAPEMIG), Research Support Foundation of the State of Rio de Janeiro (FAPERJ) and Brazilian Council for Scientific and Technological Development (CNPq).

References

Bruyn, D. De, et al., 2007. From Myrrha to Xt-ads: the Design Evolution of an Experimental Ads System, pp. 848–854.
Espinosa-Paredes, G., Álvarez-Ramírez, J., Vázquez, A., 2006. Detecting long-range correlation with detrended fluctuation analysis: application to BWR stability. Ann. Nucl. Energy 33 (16), 1309–1314.
Espinosa-Paredes, G., Polo-Labarrios, M.-A., Espinosa-Martínez, E.-G., 2011. Fractional neutron point kinetics equations for nuclear reactor dynamics. Ann. Nucl. Energy 38 (2–3), 307–330. Available at: http://dx.doi.org/10.1016/j.anucene.2010.10.012.
Hahn, M.G. et al., 2010. On time-changed Gaussian processes and their associated Fokker-Planck-Kolmogorov equations. Arxiv preprint arXiv10112473, 16, p.13. Available at: http://arxiv.org/abs/1011.2473.
Hurst, H.E., et al., 1966. Long-term storage: an experimental study. J. R. Stat. Soc. Ser. A General. 129 (4), 591. Available at: http://www.jstor.org/stable/10.2307/2982267?origin=crossref.
Koutsoyiannis, D., Paschalis, A., Theodoratos, N., 2011. Two-dimensional Hurst-Kolmogorov process and its application to rainfall fields. J. Hydrol. 398 (1–2), 91–100. Available at: http://dx.doi.org/10.1016/j.jhydrol.2010.12.012.
Li, W.L., et al., 2013. Flow regime identification in a three-phase bubble column based on statistical, Hurst, Hilbert-Huang transform and Shannon entropy analysis. Chem. Eng. Sci. 102, 474–485. Available at: http://dx.doi.org/10.1016/j.ces.2013.08.052.
Mukaiyama, T., et al., 2001. Review of research and development of accelerator-driven system in Japan for transmutation of long-lived nuclides. Prog. Nucl. Energy 38 (1–2), 107–134.
Peng, C.K., et al., 1994. Mosaic organization of DNA nucleotides. Phys. Rev. E 49 (2), 1685–1689.
Sánchez, M.Á., et al., 2015. The effect of the underlying distribution in hurst

exponent estimation. PLoS One 10 (5), 1–18.
Suzuki, T., et al., 2005. Transient analyses for accelerator driven system PDS-XADS using the extended SIMMER-III code. Nucl. Eng. Des. 235 (24), 2594–2611.
Vandeplassche, D., et al., 2011. The MYRRHA linear accelerator. Work 2718–2720.
Vyawahare, V.A., Nataraj, P.S.V., 2013. Fractional-order modeling of neutron transport in a nuclear reactor. Appl. Math. Model. 37 (23), 9747–9767. Available at: http://dx.doi.org/10.1016/j.apm.2013.05.023.
Weron, R., 2015. DFA: MATLAB function to compute the Hurst exponent using Detrended Fluctuation Analysis (DFA). IDEAS. Available at: https://ideas.repec.org/c/wuu/hscode/m11002.html (Accessed 9 September 2015).