ILLINOIS INSTITUTE OF TECHNOLOGY

DOUBLE LINE-to-NEUTRAL FAULTS WITH UNEQUAL FAULT IMPEDANCES

BY

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PREFACE

In this thesis component methods are used to study the double-line-to-neutral fault with different fault impedances. Both analytical and a-c network calculator techniques have been developed and a comparison between the various approaches to the problem has been presented.

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CHAPTER I

INTRODUCTION

Short circuit studies in three-phase power systems are commonly made for relay and circuit breaker applications and as such they should be fairly accurate. The role of the fault impedances must not be overlooked if an adequate setting of the protective scheme is to be achieved. Methods of handling this problem are available and essentially they consist of establishing a linear transformation between two sets of variables, one being a set of system voltages and currents and the other being a set of component voltages and currents. System voltages and currents are functions of time and in general the so called instantaneous values shoud be used, giving rise to an approach involving differential equations, derived from Kirchhoff's laws and capable of describing both transient and steady states. Because steady state may be directly studied by phasor quantities, that are much easier to handle than differential equations, and because steady-state analysis has sufficed, for most purposes, in power systems practice, it has become quite common to use phasor transformations in the study of system unbalances.

The usefulness and simplicity of the transformation approach is that single-phase independent networks can be derived from the actual system, in which component voltages establish the flow of component currents and that by simply interconnecting these networks, according to specified rules, one can represent many different types of system unbalances, in a manner appropriate to a-c network calculator usage. Also numerical computation is in general considerably facilitated by the use of the same methods, but a-c calculating boards have been given special consideration in handling the problems of the extensively interconnected power systems of the present time.

Among the transformation methods, the one that has been commonly used is symmetrical components. Faults that are symmetrical with respect to the reference phase may be very conveniently studied by this method. Interconnections between sequence networks to represent such faults have been worked out,[1*] and are very well known to system engineers. Nevertheless, certain dissymmetries may give rise to non reciprocal mutual impedances coupling the sequence networks that, even though offering no mathematical difficulty, are not physically realizable. Perhaps, because of this difficulty and because the symmetrical component method has been so popular, the study of double line-to-neutral faults has been restricted to equal impedances at the point of fault. It seems that such restriction has not been a serious inconvenience, but it must be realized that continuous improvement in electrical equipment calls for the refinement of techniques for determination of system performance, in order to apply those new facilities more effectively and take full advantage of their improved characteristics. Thus it is desirable to refine certain technical procedures that have been used for years and, among them, investigation of the influence on system quantities of different fault impedances in a double line-to-neutral fault has been selected for this thesis. As a first approach, symmetrical components have been used and a solution that seems quite in line with a-c network calculator practice has been obtained.

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* For numbered references, see Bibliography.

At present, the study of simultaneous faults is not regularly included in the determination of system performance, but such considerations may become increasingly important. To allow for this possibility, the application of alpha, beta and zero components and criss, cross and zero components to the problem has been studied. The use of such components considerably facilitates the study of faults occuring at the same time, in different locations throughout the system, besides providing a possibility of interconnection of the component networks to correctly represent the fault, regardless of questions of symmetry with respect to the reference phase or equality of fault impedances. Also, analytical solution of power system problems by means of these components requires less work than the use of symmetrical components. The possibility of interconnection of the component networks to represent the fault in cases where symmetrical components will not provide the same facility, together with the fact that α, β, 0 components and u, v, 0 components are a natural description of power system quantities, recomend the use of these components as a means of visualizing the physical implications of a problem. All of these characteristics of the various types of components here concerned will be noticeable in the solutions that are presented for the double line-to-neutral fault, with different fault impedances, and will in a sense confirm general conclusions that have been drawn in previous works as to the merits of each one.

CHAPTER II

SYMMETRICAL COMPONENTS APPROACH

General

Consider a small section AB of a three phase power system, consisting of phases a, b, c, and neutral and an arrangement of impedances Z_a, Z_f and Z_n, as shown in Fig. 1. Connecting terminals 2 to y, 3 to z, letting terminal 1 remain open circuited, will represent a fault between phases b and c and neutral, with fault impedances $Z_a + Z_f$ on phase b and Z_a on phase c, and with impedance Z_n in the neutral. As the impedances Z_a, Z_f and Z_n may assume any values, the fault impedances $Z_a + Z_f$ and Z_a and the neutral impedance Z_n may be adjusted to satisfy any desired conditions. To keep Z_f with positive resistance, which is desirable for a-c network calculator representation, it may, whenever necessary, be shifted to branch z, leaving y short circuited to n, or what is the same thing, designations y and z may be interchanged. Performing all possible connections of terminals y and z to any pair of terminals chosen among terminals 1, 2 and 3, produces six different types of connections, each one representing a double line-to-neutral fault with different fault impedances.

The sequence networks may now be derived from the system diagram, the impedance Z_a being included in the positive, negative and zero sequence networks and the impedance Z_n being shown as $3Z_n$ in the zero sequence network. The fault terminals in the sequence networks will correspond to points 1, 2, 3 and n in the system diagram and the fault impedance to be considered in the analysis that follows will be Z_f, without any loss of generality whatsoever. In this regard, the impedances

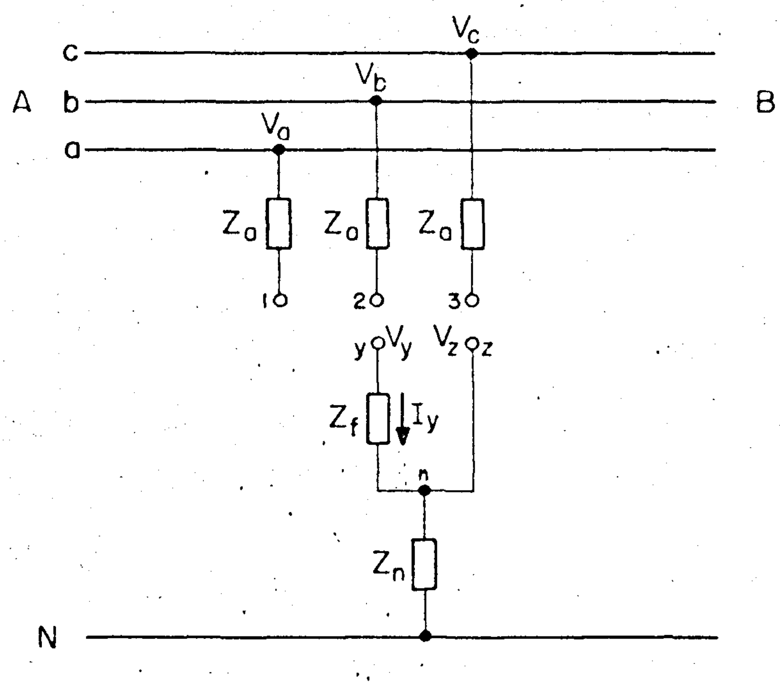

Fig. 1. **Section of a Three Phase Power System with Arrangement of Impedances to Represent the Fault.**

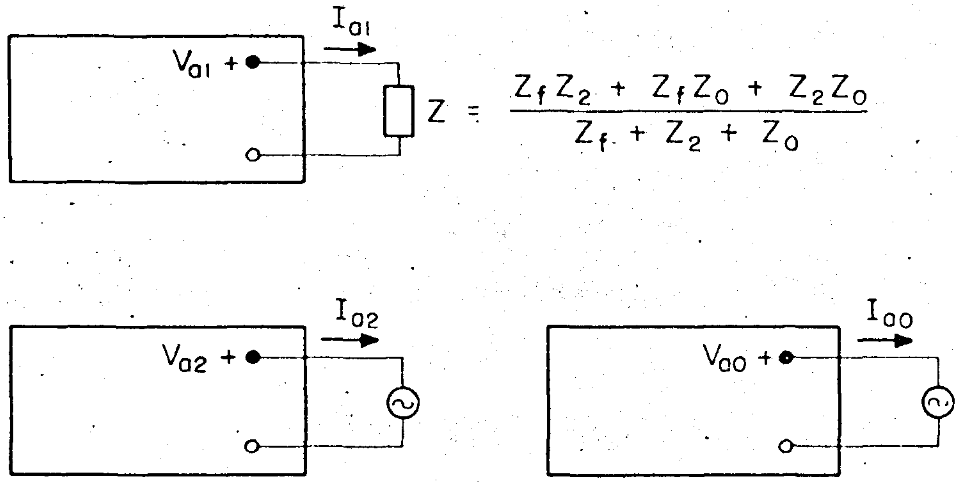

Fig. 2. **Sequence Networks Connections for the Determination of Sequence Quantities Distribution.**

Z_a are not an integrated part of the system, but they are considered as such in order to achieve analytical simplification.

The fault representations, in the system diagram, for all six possible cases above mentioned and with the conventions as to fault terminals, are shown in Fig. 3.

Determination of Sequence Quantity Distribution

It is proved in the appendix 1 that, with the foregoing conventions, the fault, in any case, may be represented by connecting to the positive sequence network an impedance(Fig. 2.)

$$Z = \frac{Z_f Z_2 + Z_f Z_0 + Z_2 Z_0}{Z_f + Z_2 + Z_0} \qquad (1)$$

Here Z_2 and Z_0 are the negative and zero sequence impedances, as viewed from the fault, and Z_f is the fault impedance already defined. The positive sequence network may then be set up on the a-c network calculator, the impedance Z connected across its fault terminals and the distreibution of positive sequence currents and voltages, throughout the network, may be determined by merely metering their values. In particular, the positive sequence currente at the fault, hereafter designated by I_{a1}, will be used in determining the distribution of negative and zero sequence currents and voltages, as will now be explained.

The negative and zero sequence currents at the fault will be called I_{a2} and I_{a0} and the ratios I_{a2}/I_{a1} and I_{a0}/I_{a1} will be designated by K_{c2} and K_{c0}, respectively. The values of K_{c2} and K_{c0} are given in table 1 and are derived in appendix 1.

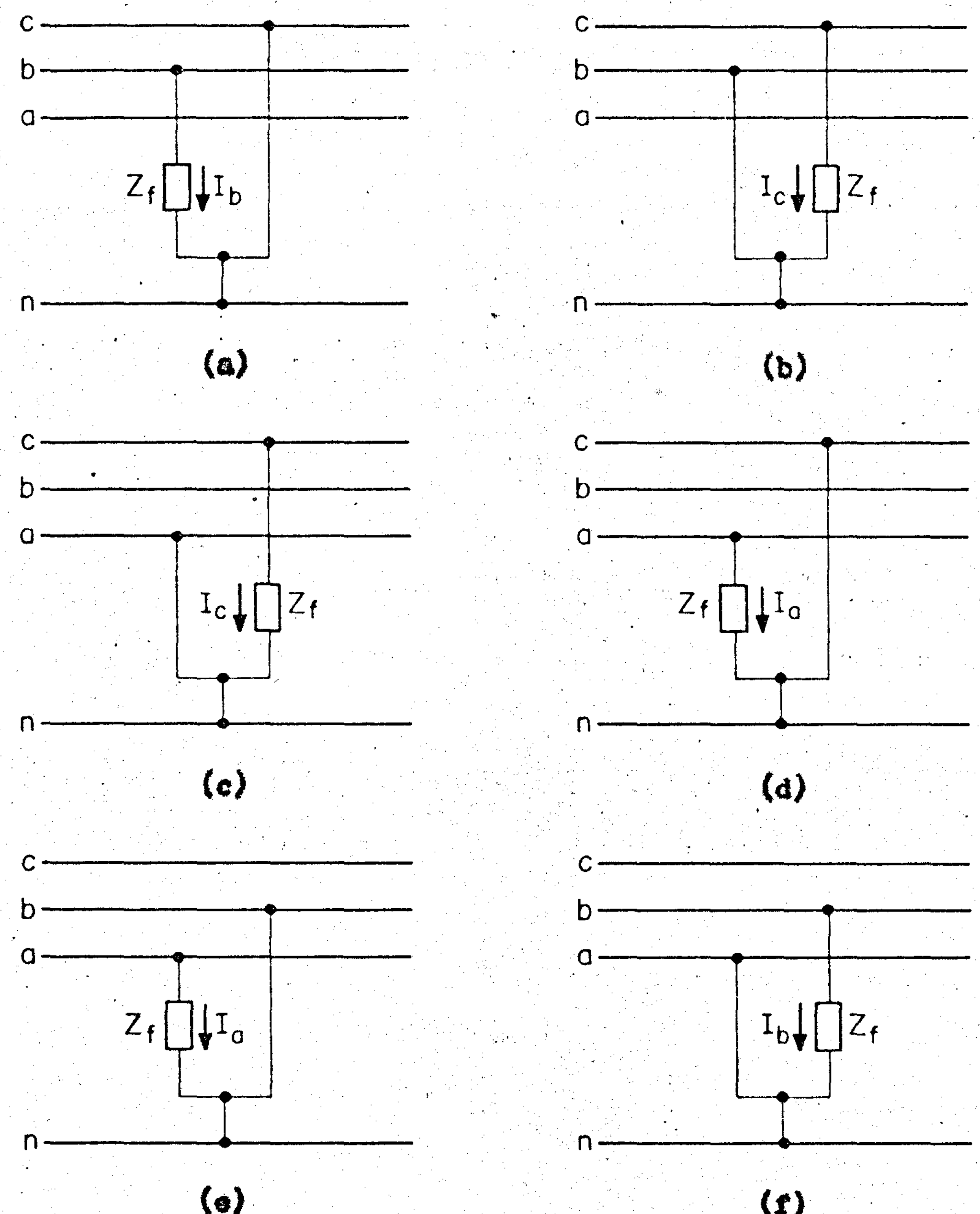

Fig. 3. Fault Representations in the System Diagram.

Table 1. Relations between Positive, Negative and Zero Sequence Currents at the Fault.

Fault on phases	Fault impedance on phase	$K_{c2} = \frac{I_{a2}}{I_{a1}}$	$K_{c0} = \frac{I_{a0}}{I_{a1}}$
bc	b	$\frac{a^2 Z_f - Z_0}{Z_f + Z_2 + Z_0}$	$\frac{a Z_f - Z_2}{Z_f + Z_2 + Z_0}$
	c	$\frac{a Z_f - Z_0}{Z_f + Z_2 + Z_0}$	$\frac{a^2 Z_f - Z_2}{Z_f + Z_2 + Z_0}$
ca	c	$\frac{Z_f - a Z_0}{Z_f + Z_2 + Z_0}$	$\frac{Z_f - a^2 Z_2}{Z_f + Z_2 + Z_0}$
	a	$\frac{a^2 Z_f - a Z_0}{Z_f + Z_2 + Z_0}$	$\frac{a Z_f - a^2 Z_2}{Z_f + Z_2 + Z_0}$
ab	a	$\frac{a Z_f - a^2 Z_0}{Z_f + Z_2 + Z_0}$	$\frac{a^2 Z_f - a Z_2}{Z_f + Z_2 + Z_0}$
	b	$\frac{Z_f - a^2 Z_0}{Z_f + Z_2 + Z_0}$	$\frac{Z_f - a Z_2}{Z_f + Z_2 + Z_0}$

By the definition of K_{c2} and K_{c0}:

$$I_{a2} = K_{c2} I_{a1} \qquad (2)$$
$$I_{a0} = K_{c0} I_{a1}$$

The negative and zero sequence voltages at the fault are given in terms of the negative and zero sequence currents and impedances by

$$V_{a2} = - Z_2 I_{a2} \qquad (3)$$
$$V_{a0} = - Z_0 I_{a0}$$

or in terms of the positive sequence currents, after Eq. (2) are substituted in Eq. (3):

$$V_{a2} = - K_{c2} Z_2 I_{a1} \qquad (4)$$
$$V_{a0} = - K_{c0} Z_0 I_{a1}$$

The most convenient way to determine the distribution of negative and zero sequence currents and voltages is perhaps to measure the correspondent distribution factors. This may be done by separately energizing the negative and zero sequence networks, at the point of fault.(fig.2). When $I_{a2} = 1$ ampere, the current measured at any location in the negative sequence network will be the distribution factor K_{d2} for that location. When $I_{a0} = 1$ ampere, the current measured at any location in the zero sequence network will be the distribution factor K_{d0} for that location. If I_2 and I_0 are the negative and zero sequence currents at a given point of the faulted system, and K_{d2} and K_{d0} the distribution factors for the correspondent points in the negative and zero sequence networks, then by definition:

$$I_2 = K_{d2} I_{a2} \qquad (5)$$
$$I_0 = K_{d0} I_{a0}$$

or after Eq. 2 are substituted in Eq. 5:

$$I_2 = K_{c2}\,K_{d2}\,I_{a1}$$
$$I_0 = K_{c0}\,K_{d0}\,I_{a1} \qquad (6)$$

Distribution factors for negative and zero sequence voltages will be called K'_{d2} and K'_{d0}, respectively. If V_2 and V_0 are the negative and zero sequence voltages, at a given point of the faulted system, and K'_{d2} and K'_{d0} the distribution factors for the correspondent points in the negative and zero sequence networks, then by definition:

$$V_2 = K'_{d2}\,V_{a2}$$
$$V_0 = K'_{d0}\,V_{a0} \qquad (7)$$

or after Eq. (4) are substituted in Eq. (7).

$$V_2 = -K_{c2}\,K'_{d2}\,Z_2\,I_{a1}$$
$$V_0 = -K_{c0}\,K'_{d0}\,Z_0\,I_{a1} \qquad (8)$$

<u>Outline of a-c Network Calculator Procedure</u>

Briefly, the procedure to be recommended for the determination of the symmetrical components of system quantities, in a double line to neutral fault with different fault impedances, using an a-c network calculator, is:

1. Set up the zero sequence network and measure Z_0 and the distribution factors of subscript zero.
2. Set up the negative sequence network and measure Z_2 and the distribution factors of subscript two.
3. Set up the positive sequence network, connect the impedance Z, given by Eq. (1), across its fault terminals, measure I_{a1} and the distribution of positive sequence quantities.

This will not in general require much more time than other short circuit studies commonly made in an a-c network calculator, because distribution factors for the zero sequence network are regularly needed, unless there are available enough line units to set up the sequence networks simultaneously and interconnect them according to the type of fault.

Eq. (6) and Eq. (8) may be applied after all board work has been finished and will provide a means to determine the distribution of negative and zero sequence quantities, in terms of the positive sequence current at the fault I_{a1}, and the factors already defined. In particular, the factors K_{o2} and K_{oo} are chosen from table 1 for the position of the fault with respect to the reference phase.

When the phase angles between the various source voltages are not of prime importance, it is customary, in a short circuit study, to consider all generated internal voltages equal in phase and magnitude. With this assumption, the positive sequence network may be represented by a generator in series with a passive network. If shunt elements are neglected and Z_2 may be considered equal to Z_1, some important simplifications are achieved. In this regard, the positive sequence network, when positive sequence quantities are being measured, (Fig.4) may be compared with the negative sequence network, when distribution factors are being determined, (Fig.5). The distribution of currents and voltages is the same in both networks, but is must be observed that sinse the point S in the negative sequence network is short circuited to neutral, the voltage V_2, at a point P, in that network, is actually measured with respect to S as reference. This implies that the voltage of the corresponding point in the positive sequence network

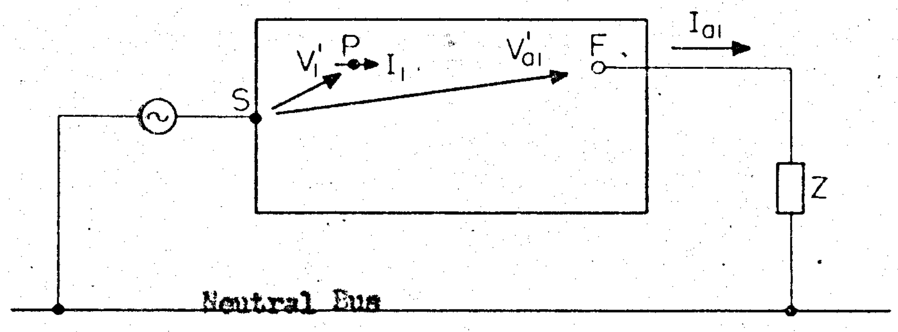

Fig. 4. Measurement of Positive Sequence Quantities in the Positive Sequence Network.

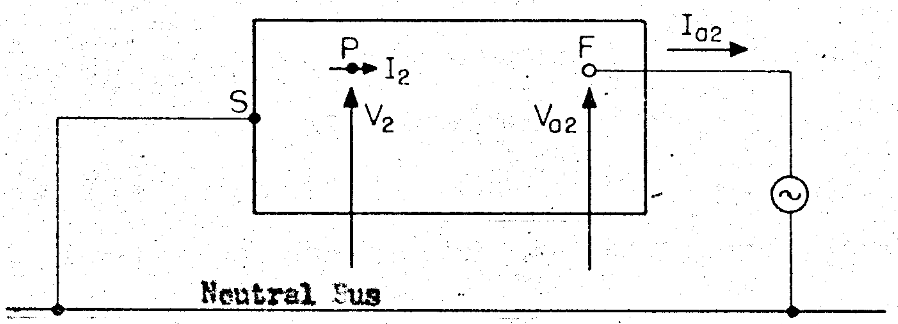

Fig. 5. Determination of Distribution Factors in the Negative Sequence Network.

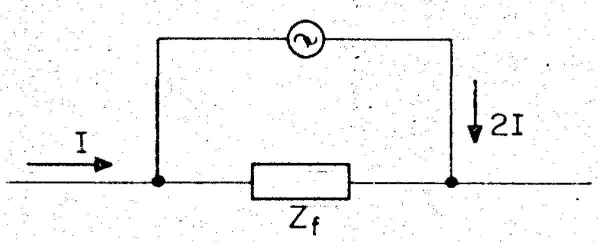

Fig. 6. Representation of a Negative Impedance in the A-C Network Calculator.

is proportional to V_2 when measured with respect to the point S of this network. With the notation indicated in Figs. 4 and 5 it follows that

$$\frac{I_2}{I_{a2}} = \frac{I_1}{I_{a1}} \quad \text{and} \quad \frac{V_2}{V_{a2}} = \frac{V'_1}{V'_{a1}}$$

from which

$$I_2 = \frac{I_{a2}}{I_{a1}} I_1 = K_{c2} I_1 \tag{6a}$$

$$V_2 = \frac{V_{a2}}{V'_{a1}} V'_1 = -\frac{Z_2 I_{a2}}{V'_{a1}} V'_1 = -K_{c2} Z_2 \frac{V'_1}{V'_{a1}} I_{a1} \tag{8a}$$

Under these circumstances, the negative sequence network need not be set up and negative sequence currents are easily computed by Eq. (6a), after positive sequence currents have been measured.

It should be observed that ordinarily voltage distribution is not required in a short circuit study. Nevertheless, voltage distribution factors, Eq. (8) and Eq. (8a) have been derived. This last equation may be used in the special case when it is desired to determine a voltage in a negative sequence network that was not set up for the short circuit study.

CHAPTER III

ALPHA, BETA AND ZERO COMPONENTS APPROACH

General

Phase voltages and currents are expressed in terms of their α, β, and 0 components as follows:[2]

$$
\begin{aligned}
V_a &= V_\alpha + V_0 \\
V_b &= -\tfrac{1}{2} V_\alpha + \tfrac{\sqrt{3}}{2} V_\beta + V_0 \\
V_c &= -\tfrac{1}{2} V_\alpha - \tfrac{\sqrt{3}}{2} V_\beta + V_0 \\
I_a &= I_\alpha + I_0 \\
I_b &= -\tfrac{1}{2} I_\alpha + \tfrac{\sqrt{3}}{2} I_\beta + I_0 \\
I_c &= -\tfrac{1}{2} I_\alpha - \tfrac{\sqrt{3}}{2} I_\beta + I_0
\end{aligned}
\qquad (9)
$$

The simultaneous solution of these equations will show that α, β, and 0 components are expressed in terms of phase quantities as follows:[2]

$$
\begin{aligned}
V_\alpha &= \tfrac{2}{3}\left(V_a - \frac{V_b + V_c}{2}\right) \\
V_\beta &= \tfrac{1}{\sqrt{3}}\left(V_b - V_c\right) \\
V_0 &= \tfrac{1}{3}\left(V_a + V_b + V_c\right) \\
I_\alpha &= \tfrac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) \\
I_\beta &= \tfrac{1}{\sqrt{3}}\left(I_b - I_c\right) \\
I_0 &= \tfrac{1}{3}\left(I_a + I_b + I_c\right)
\end{aligned}
\qquad (10)
$$

The use of Eq. (9) and Eq. (10) together, facilitates the transformation of an equation involving phase quantities into the corresponding equation envolving the α, β, and 0 components of those phase quantities.

Transforming system quantities into α, β, and 0 components corresponds to transforming a three-phase network into three single-phase component networks, called the α network, the β network and the zero network, whose impedances are the same as the sequence impedances used in symmetrical components,[2] for the common assumption of symmetrical system and $Z_2 = Z_1$. Under these circumstances and for balanced system condition, the component networks are independent and so the flow of component currents is not related. Zero component currents cannot flow in this case, because there are no generated zero component voltages under balanced condition.

When a fault occurs somewhere in the system, the balanced conditions is upset and an interconnection of the component networks, at the point of fault, is made to represent the fault, according to the equations describing it. Such equations are first written in terms of phase voltages and currents, at the fault, and there are three for each fault location. They are then expressed in α, β, and 0 components by means of Eq. (9) and Eq. (10) and translated into appropriate interconnections of the component networks.

Discussion of the Problem

In this case, fig. 3 provide a schematic diagram of the conditions to be studied and contain all possibilities that may be met with in a double line-to-neutral fault, with different fault impedances, as has been pointed out, in connection with the same figure, when the symmetrical components approach was discussed. The same conventions made in chapter II, as to fault impedances, are maintained, as they are equally useful here. In appendix 2 is given the derivation of the equations describing the fault, in terms of α, β, and 0 components, for the fundamental cases among those shown in Fig. 3. Derivations that would be entirely similar to the ones just mentioned have been omitted, but all results are given in table 2. Interconnections of the α, β, and 0 networks to represent the fault have been worked out, according to the equations in table 2 and are shown in Fig. 7 to Fig. 14.

Analysis of Proposed Solutions

It may be noticed that in the circuits of Fig. 10 and Fig. 12, the impedance $-Z_f$ is shown. Since Z_f has been chosen with a positive resistance, the representation of $-Z_f$ in an a-c network calculator will entail some complication. To simulate a negative impedance $-Z_f$, a generator is connected across the terminals of $+Z_f$ (Fig. 6) and adjusted to supply a current opposite to the normal current I flowing through $-Z_f$ and of double magnitude. In some instances, it may be practical to absorb $-Z_f$ or its resistance component in the impedance of the β network and so avoid the complications mentioned above. Another procedure that may be followed is to use the alternative connections of Fig. 11 and Fig. 13 that provide an elegant solution, in which no negative impedance is required. When other conditions are immaterial, the choice between the two types of connections may be set by the equipment available, since one uses three impedances and one mutual transformer and the other uses three mutual transformers and one impedance.

In all the proposed diagrams of interconnections, mutual transformers were used rather than modification of impedances, voltages and currents of the component networks. When simultaneous unbalances are

Table 2. Relations between Alpha, Beta and Zero Components of Voltages and Currents at the Fault.

Number	Fault on phases	Fault impedance on phase	Describing equations
1	bc	b	$V_\alpha - 2V_0 = \frac{3}{2} Z_f I_\alpha - \frac{\sqrt{3}}{2} Z_f I_\beta$ $V_\beta = \frac{1}{2} Z_f I_\beta - \frac{\sqrt{3}}{2} Z_f I_\alpha$ $I_\alpha + I_0 = 0$
2		c	$V_\alpha - 2 V_0 = \frac{3}{2} Z_f I_\alpha + \frac{\sqrt{3}}{2} Z_f I_\beta$ $V_\beta = \frac{\sqrt{3}}{2} Z_f I_\alpha + \frac{1}{2} Z_f I_\beta$ $I_\alpha + I_0 = 0$
3	ca	c	$V_\alpha + V_0 = 0$ $\frac{V_\beta}{\sqrt{3}} - V_0 = \frac{2}{\sqrt{3}} Z_f I_\beta$ $2 I_0 = I_\alpha - \sqrt{3} I_\beta$
4		a	$V_\alpha + \frac{V_\beta}{\sqrt{3}} = Z_f I_\alpha - Z_f \frac{I_\beta}{\sqrt{3}}$ $V_0 - \frac{V_\beta}{\sqrt{3}} = Z_f I_0 + Z_f \frac{I_\beta}{\sqrt{3}}$ $2 I_0 = I_\alpha - \sqrt{3} I_\beta$

Table 2. (Continued)

Number	Fault on Phases	Fault impedance on phase	Describing equations
			(Alternative solution)
4a	ca	a	$2 V_0 = V_\alpha + \sqrt{3} V_\beta$
			$\sqrt{3} V_\alpha + V_\beta = 2 Z_f \left(\frac{I_\alpha}{\sqrt{3}} + \frac{I_0}{\sqrt{3}} \right)$
			$2 I_0 = I_\alpha - \sqrt{3} I_\beta$
5	ab	a	$V_\alpha - \frac{V_\beta}{\sqrt{3}} = Z_f I_\alpha + Z_f \frac{I_\beta}{\sqrt{3}}$
			$V_\beta + \sqrt{3} V_0 = \frac{\sqrt{3}}{2} Z_f I_\alpha + \frac{1}{2} Z_f I_\beta$
			$2 I_0 = I_\alpha + \sqrt{3} I_\beta$
			(Alternative solution)
5a		a	$2 V_0 = V_\alpha - \sqrt{3} V_\beta$
			$\sqrt{3} V_\alpha - V_\beta = 2 Z_f \left(\frac{I_\alpha}{\sqrt{3}} + \frac{I_0}{\sqrt{3}} \right)$
			$2 I_0 = I_\alpha + \sqrt{3} I_\beta$
6		b	$V_\alpha + V_0 = 0$
			$\frac{V_\beta}{\sqrt{3}} + V_0 = \frac{2}{\sqrt{3}} Z_f I_\beta$
			$2 I_0 = I_\alpha + \sqrt{3} I_\beta$

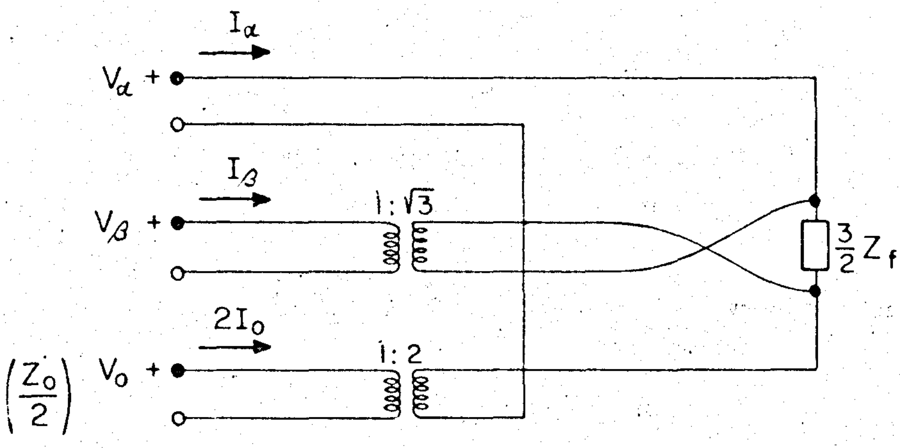

Fig. 7. Fault b- c- n. Fault Impedance on Phase b.

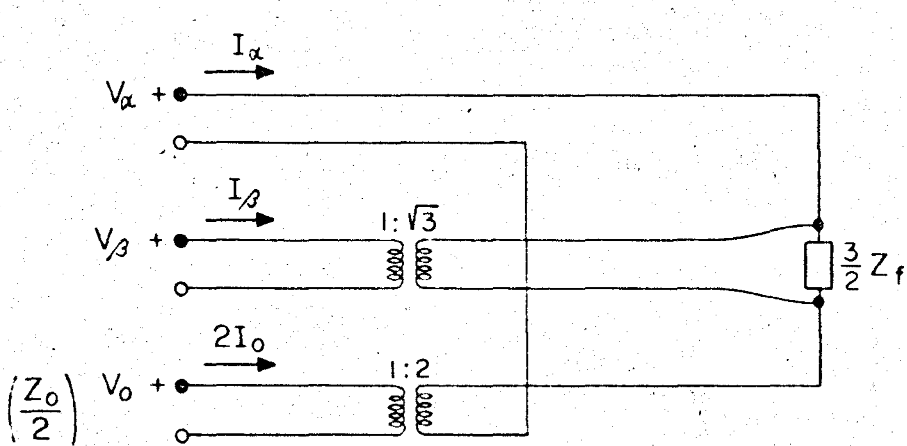

Fig. 8. Fault b- c- n. Fault Impedance on Phase c.

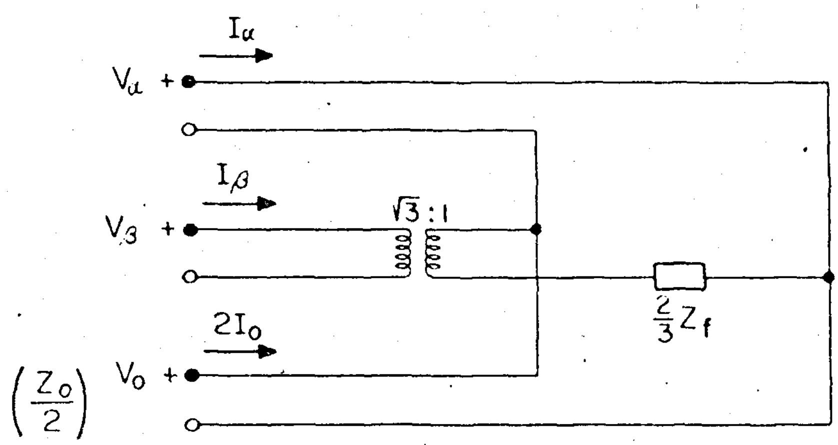

Fig. 9. Fault c- a- n. Fault Impedance on Phase c.

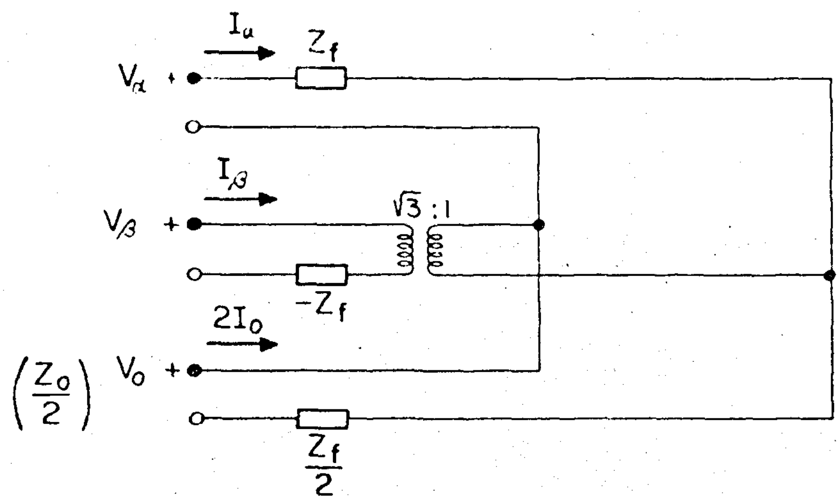

Fig. 10. Fault c- a- n. Fault Impedance on Phase a.

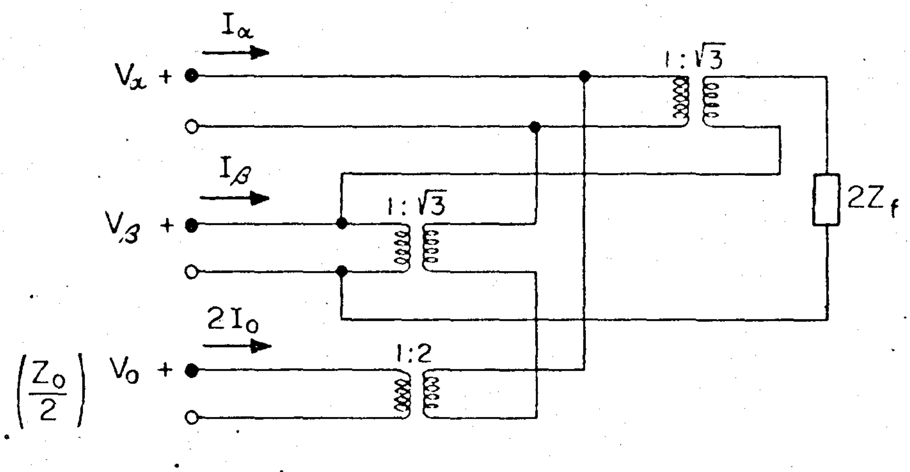

Fig. 11. Fault c-a-n. Fault Impedance on Phase a. (Alternate Solution).

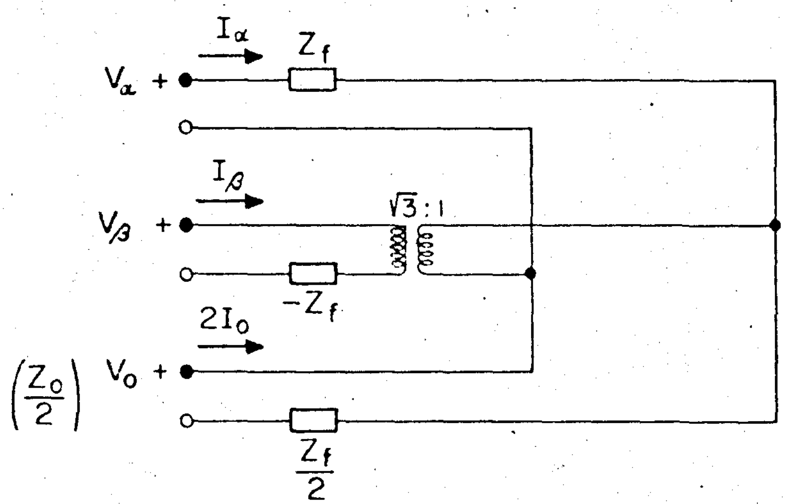

Fig. 12. Fault a-b-n. Fault Impedance on Phase a.

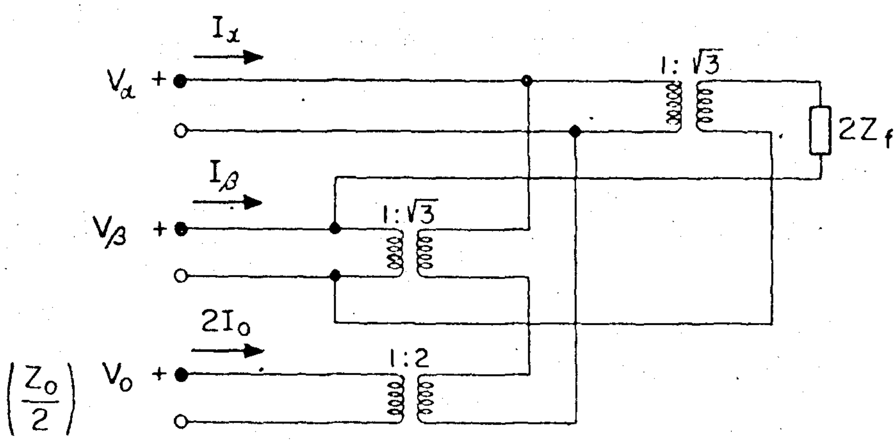

Fig. 13. Fault a-b-n. Fault Impedance on Phase a. (Alternate Solution).

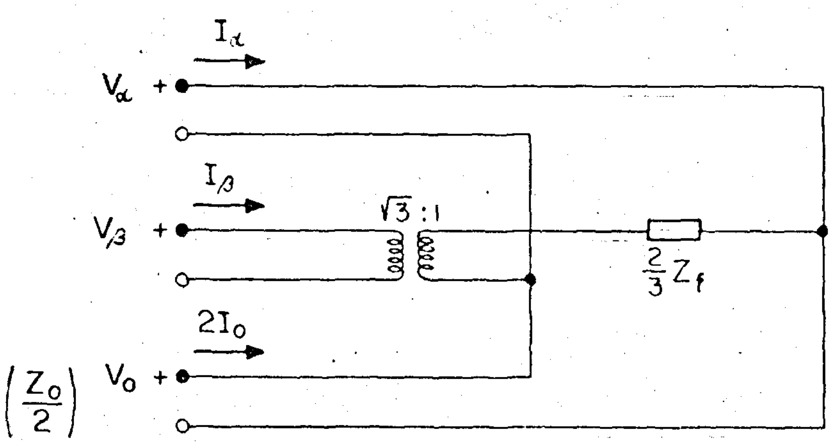

Fig. 14. Fault a-b-n. Fault Impedance on Phase b.

being considered,[2] this has the advantage of avoiding the introduction of restrictions that may not be necessarily true in the system, and for this reason the mutual transformers are sometimes called isolating transformers.

We also notice that the impedance of the zero network appears as $Z_0/2$ in all the proposed diagrams, as is usual for other types of faults studied elsewhere.[2]

CHAPTER IV

CRISS CROSS AND ZERO COMPONENTS APPROACH

<u>General</u>

Phase voltages and currents are expressed in terms of their u, v, and 0 components as follows:[3]

$$V_a = \sqrt{2}\, V_u + V_0$$

$$V_b = -\frac{V_u}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} V_v + V_0 \qquad (11)$$

$$V_c = -\frac{V_u}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}} V_v + V_0$$

$$I_a = \sqrt{2}\, I_u + I_0$$

$$I_b = -\frac{I_u}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} I_v + I_0$$

$$I_c = -\frac{I_u}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}} I_v + I_0$$

The simultaneous solution of these equations will show that u, v, and 0 components are expressed in terms of phase quantities as follows:[3]

$$V_u = \frac{1}{3\sqrt{2}} \left(2 V_a - V_b - V_c \right)$$

$$V_v = \frac{1}{\sqrt{3}\sqrt{2}} \left(V_b - V_c \right)$$

$$V_0 = \frac{1}{3} \left(V_a + V_b + V_c \right) \qquad (12)$$

$$I_u = \frac{1}{3\sqrt{2}} \left(2 I_a - I_b - I_c \right)$$

$$I_v = \frac{1}{\sqrt{3}\sqrt{2}} \left(I_b - I_c \right)$$

$$I_0 = \frac{1}{3} \left(I_a + I_b + I_c \right)$$

The use of Eq. (11) and Eq. (12) together facilitates the transformation of an equation involving phase quantities into the corresponding equation involving the u, v, and 0 components of these phase quantities.

Most of the properties concerning α, β, and 0 components are also applicable to u, v, and 0 components.[3] Briefly, the transformation here concerned is also associated with three component networks, called the criss network, the cross network and the zero network, with impedances equal to the sequence impedances Z_1, Z_2, and Z_0, respectively, for the common assumption of symmetrical system and $Z_2 = Z_1$. The component networks, under balanced conditions, are then independent, and as both criss and cross components are needed to represent positive sequence quantities, only the zero network is passive.

Discussion of the Problem

By following the procedure used in chapter III, the equations describing the fault have been derived in appendix II in terms of criss, cross and zero components, for the fundamental cases among those shown in Fig. 3, omitting cases whose derivation would be entirely similar to the ones just mentioned. Nevertheless, the results for all possible cases are given in table 3 and the corresponding diagrams of interconnections are shown in Fig. 15 to Fig. 22. In all these interconnections mutual transformers are used, as they are necessary to maintain the impedances of the component networks equal to the sequence impedances of symmetrical components. Keeping component impedances equal to sequence impedances is an essential feature of criss, cross and zero components, as it provides the desirable

Table 3. Relations between Criss, Cross and Zero Components of Voltages and Currents at the Fault

Number	Fault on phases	Fault impedance on phase	Describing Equations
1	bc	b	$V_u = \sqrt{2}\, V_0 = \frac{3}{2} Z_f I_u - \frac{\sqrt{3}}{2} Z_f I_v$
			$V_v = \frac{Z_f}{2} I_v - \frac{\sqrt{3}}{2} Z_f I_u$
			$I_u + \frac{I_0}{\sqrt{2}} = 0$
2	[illegible]	c	$V_u = \sqrt{2}\, V_0 = \frac{3}{2} Z_f I_u + \frac{\sqrt{3}}{2} Z_f I_v$
			$V_v = \frac{\sqrt{3}}{2} Z_f I_u + \frac{Z_f}{2} I_v$
			$I_u + \frac{I_0}{\sqrt{2}} = 0$
3	ca	c	$V_u + \frac{V_0}{\sqrt{2}} = 0$
			$\frac{V_v}{\sqrt{3}} - \frac{V_0}{\sqrt{2}} = \frac{2}{\sqrt{3}} Z_f I_v$
			$\sqrt{2}\, I_0 = I_u - \sqrt{3}\, I_v$
4	ca	a	$V_u + \frac{V_v}{\sqrt{3}} = Z_f I_u - Z_f \frac{I_v}{\sqrt{3}}$
			$\frac{V_0}{\sqrt{2}} - \frac{V_v}{\sqrt{3}} = \frac{Z_f}{\sqrt{2}} I_0 + \frac{Z_f}{\sqrt{3}} I_v$
			$\sqrt{2}\, I_0 = I_u - \sqrt{3}\, I_v$

Table 3. (Continued)

Number	Fault on phases	Fault impedance on phase	Describing equations
4a	ca	a	(Alternative solution) $\sqrt{2}\, V_0 = V_u + \sqrt{3}\, V_v$ $\sqrt{3}\, V_u + V_v = 2\, Z_f \left(\frac{I_u}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}\, I_0 \right)$ $\sqrt{2}\, I_0 = I_u - \sqrt{3}\, I_v$
5	ab	a	$V_u - \frac{V_v}{\sqrt{3}} = Z_f\, I_u + Z_f \frac{I_v}{\sqrt{3}}$ $\frac{V_0}{\sqrt{2}} + \frac{V_v}{\sqrt{3}} = \frac{Z_f}{2}\, I_u + \frac{Z_f}{2} \frac{I_v}{\sqrt{3}}$ $\sqrt{2}\, I_0 = I_u + \sqrt{3}\, I_v$
5a		a	(Alternative solution) $\sqrt{2}\, V_0 = V_u - \sqrt{3}\, V_v$ $\sqrt{3}\, V_u - V_v = 2\, Z_f \left(\frac{I_u}{\sqrt{3}} + \frac{I_0}{\sqrt{2}\,\sqrt{3}} \right)$ $\sqrt{2}\, I_0 = I_u + \sqrt{3}\, I_v$
6	ab	b	$V_u + \frac{V_0}{\sqrt{2}} = 0$ $\frac{V_0}{\sqrt{2}} + \frac{V_v}{\sqrt{3}} = \frac{2}{\sqrt{3}}\, Z_f\, I_v$ $\sqrt{2}\, I_0 = I_u + \sqrt{3}\, I_v$

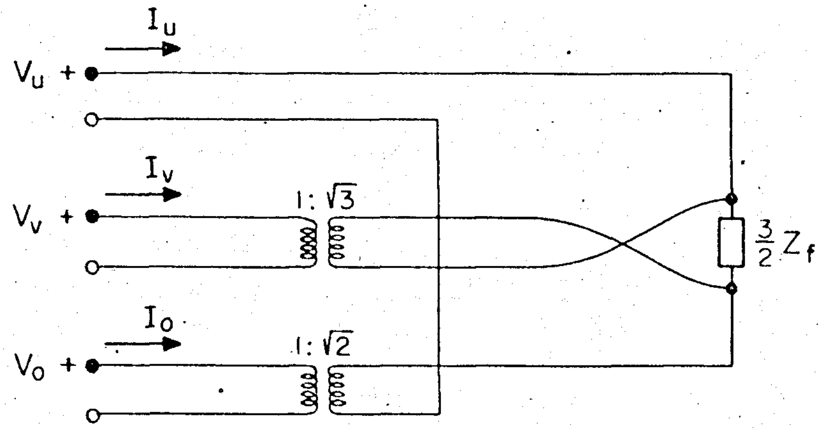

Fig. 15. Fault b-c-n. Fault Impedance on Phase b.

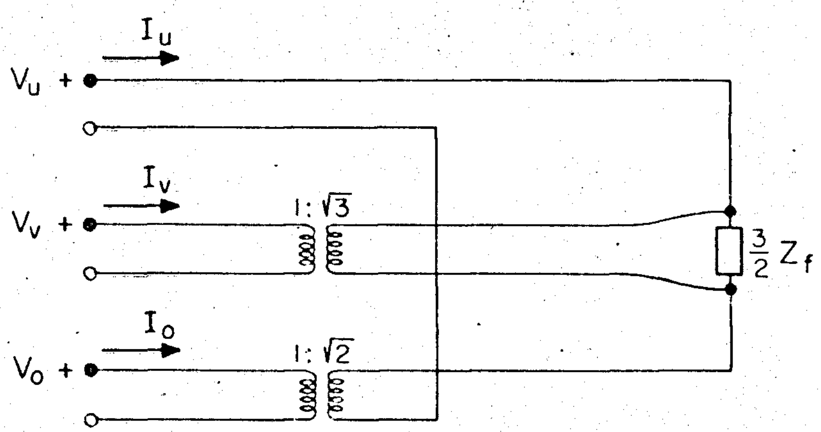

Fig. 16. Fault b-c-n. Fault Impedance on Phase c.

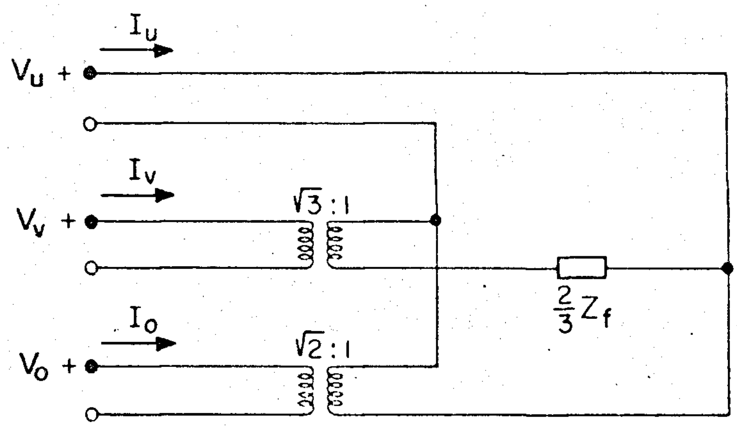

Fig. 17. Fault c-a-n. Fault Impedance on Phase c.

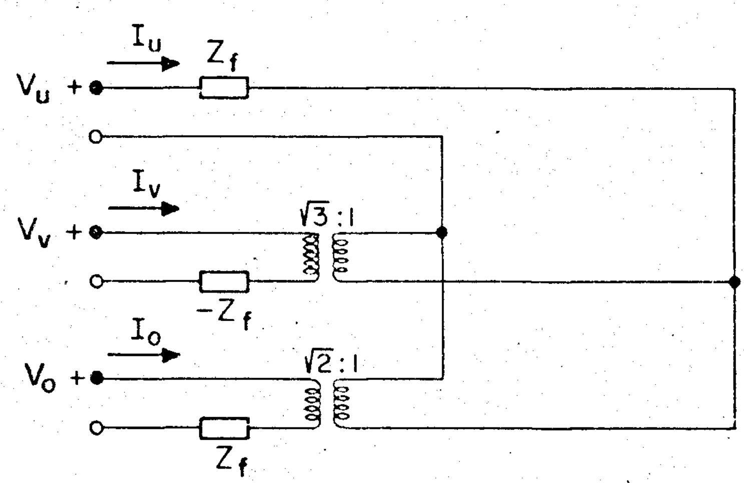

Fig. 18. Fault c-a-n. Fault Impedance on Phase a.

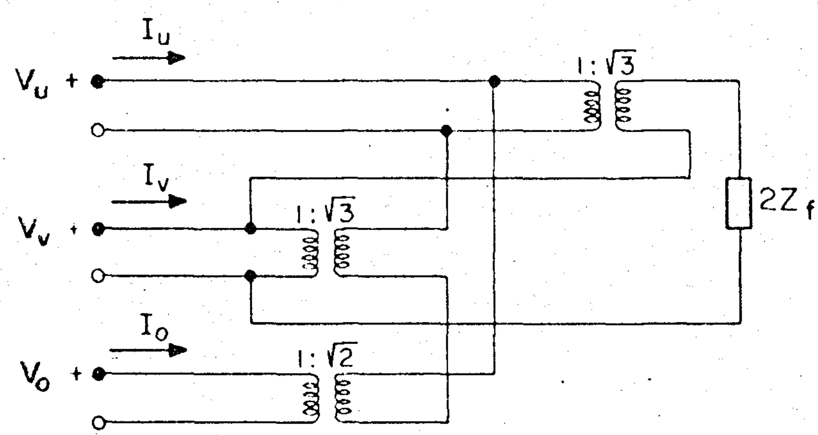

Fig. 19. Fault c-a-n. Fault Impedance on Phase a. (Alternative Solution)

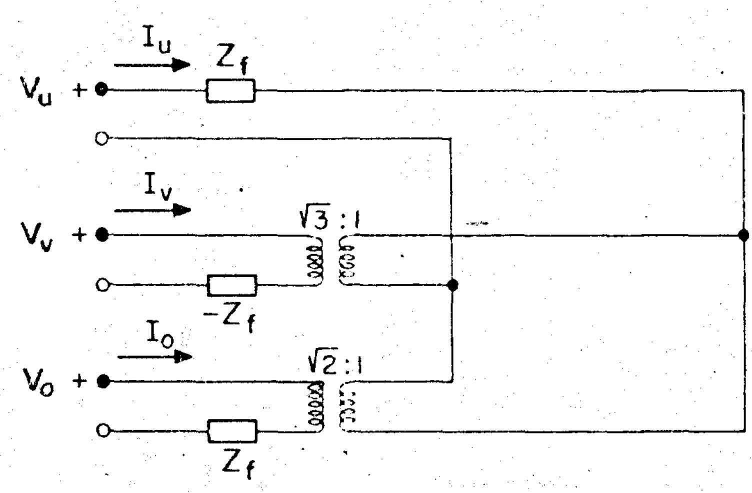

Fig. 20. Fault a-b-n. Fault Impedance on Phase a.

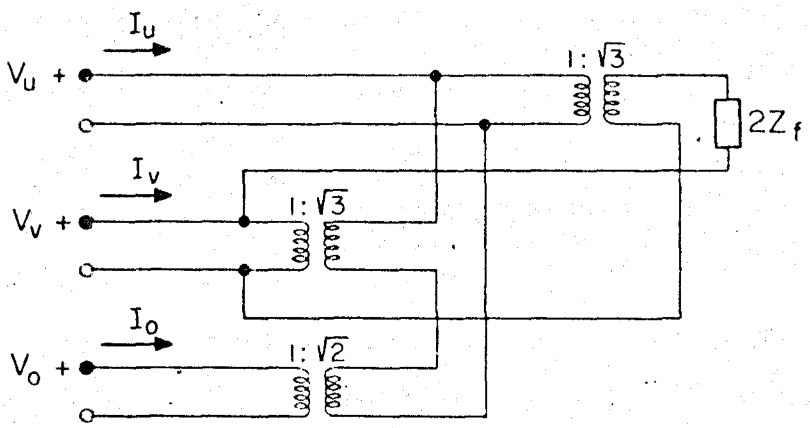

Fig. 21. Fault a-b-n. Fault Impedance on Phase a. (Alternative Solution).

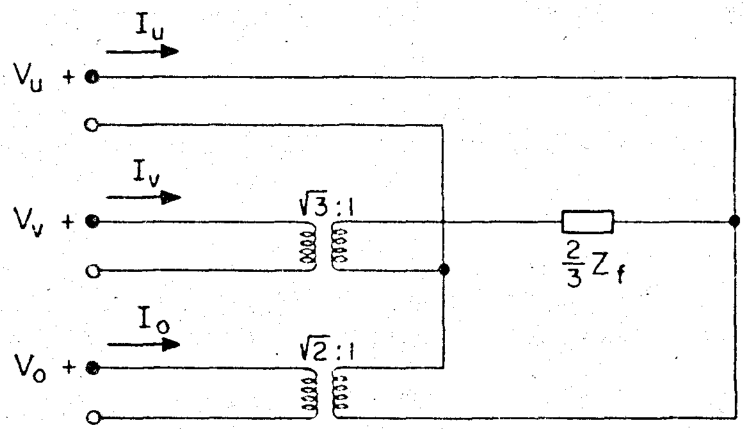

Fig. 22. Fault a-b-n. Fault Impedance on Phase b.

flexibility[3] for switching to symmetrical components, or vice versa, in an a-c network calculator, with a minimum of change. In this regard, it may be noticed that the diagrams here proposed confirm results previously found[2,3] that, whereas alpha, beta, and zero components use the zero network with impedance set at $Z_0/2$, criss, cross, and zero components will always provide diagrams with exactly the sequence impedances in each component network.

The mutual transformers act also as isolating transformers and maintain the suitability of u, v, and 0 components for the study of simultaneous faults.

Comparison Between the Various Approaches

As criss, cross and zero components, in comparison with alpha, beta and zero components, have the advantage of maintaining the impedance of the zero network at the value Z_0 and share with them all the merits, it seems reasonable that we limit the present comparison to u, v, and 0 components and symmetrical components.

The results that have been obtained with the symmetrical components method seem to fit the general procedure followed in a-c network calculators to study common types of fault. The time required for the measurement of sequence quantities will be the same as in ordinary cases and the fact that the fault impedances are different introduces only a slight complication in the factors K_{o3} and K_{c0}. Because only the positive sequence network has generated electromotive forces, symmetrical components are inherently simpler than other methods. However, when simultaneous unbalances are contemplated, criss, cross and zero components present a decided advantage and should be used.

Both the criss and the cross networks are active networks, the electromotive force in the cross network being shifted ninety degree backwards with respect to the electromotive force in the criss network, but they are equal in magnitude to 1/ 2 times the corresponding positive sequence electromotive force.

It is apparent that criss, cross and zero components require more equipment than symmetrical components for an a-c network calculator set up, but they provide interconnections of the component networks to correctly represent the fault, regardless of questions of symmetries with respect to the reference phase or equality of fault impedances, and this recommends them for the study of simultaneous unbalances.

An interesting observation[3] is that criss, cross and zero components are either in phase or in phase opposition and so their addition may be performed by means of potential and current transformers, thus making the a-c network calculator directly readable in phase quantities. Corresponding procedure with symmetrical components would not be practical, since it would require complicated phase shifters of questionable accuracy.

CHAPTER V

CONCLUSIONS

1. The study of system quantities, in a double line to neutral fault, with different fault impedances, may be made both analytically or by a-c network calculator methods, using linear transformations between phasor quantities. A-c network calculator methods are more suited to the study of modern power system and as such were given special consideration.

2. Symmetrical components may be used in a manner quite in line with present techniques. Special multipliers were derived, in conjunction with which the distribution factors may be applied in a straightforward manner, to give negative and zero sequence quantities in terms of the positive sequence current at the fault, for any combination of fault impedances and faulted phases. The use of distribution factors is an essential feature of the method here presented, but they are quite common anyway in ordinary types of short circuit studies, since there are hardly enough line units to set the sequence networks simultaneously.

3. Alpha, beta, and zero components and criss, cross, and zero components may be applied in a manner quite similar. Diagrams of interconnections of component networks, permitting a straightforward setting of the a-c network calculator, for the study of the fault, in any combination of fault impedances and faulted phases, were developed. As far as a-c network calculator techniques are concerned, these components require more equipment that the symmetrical components, but they are most suited to the study of simultaneous unbalances. Alpha

beta, and zero components require a setting of the impedance of the zero network at $Z_0/2$ and criss, cross, and zero components provide diagrams in which the impedance of the component networks are exactly the same as the impedances of the sequence networks of symmetrical components, for the common assumption of symmetrical system and $Z_2 = Z_1$. This recommends the use of criss, cross and zero components, because of the facility in changing from them to symmetrical components and vice versa, in an a-c network calculator.

APPENDIX

APPENDIX I

DERIVATION OF SYMMETRICAL COMPONENTS APPROACH

Phase voltages and currents are expressed in terms of their symmetrical components as follows:

$$
\begin{aligned}
V_a &= V_{a1} + V_{a2} + V_{a0} \\
V_b &= a^2V_{a1} + aV_{a2} + V_{a0} \\
V_c &= aV_{a1} + a^2V_{a2} + V_{a0} \\
I_a &= I_{a1} + I_{a2} + I_{a0} \\
I_b &= a^2I_{a1} + aI_{a2} + I_{a0} \\
I_c &= aI_{a1} + a^2I_{a2} + I_{a0}
\end{aligned} \qquad (13)
$$

If phases a, b, and c are made to correspond respectively to the digits 1, 2 and 3, then (Eq. 13) may be written more compactly

$$
\begin{aligned}
V_x &= a^{4-x}V_{a1} + a^{x-1}V_{a2} + V_{a0} \\
I_x &= a^{4-x}I_{a1} + a^{x-1}I_{a2} + I_{a0} \qquad (x = 1, 2, 3)
\end{aligned} \qquad (13a)
$$

In accordance with the discussion given in chapter II, the conditions of a double line-to-neutral fault, with different fault impedances, may be realized if (Fig. 1) terminals y and z are connected to any pair of terminals chosen among terminals 1, 2, and 3.

There are three different groups of connections for which Z_f will appear at the branch y, hereafter referred to as the first sequence. By interchanging designations y and z, Z_f will be shifted to branch z in each group of connections of the first sequence and the sequence so obtained will be called the second sequence.

Calling x the terminal that will conduct no current, the following connections may be made to represent the fault in any conditions:

For $x = 1$, connect y to 2 and z to 3.

For $x = 2$, connect y to 3 and z to 1.

For $x = 3$, connect y to 1 and z to 2.

The equations describing the fault in phase quantities are

$$I_x = 0$$

$$V_y = Z_f I_y \qquad (14)$$

$$V_z = 0$$

Expressing Eq. (14) in symmetrical components gives:

$$a^{4-x} I_{a1} + a^{x-1} I_{a2} + I_{a0} = 0 \qquad (15)$$

$$a^{4-y} V_{a1} + a^{y-1} V_{a2} + V_{a0} = Z_f \left(a^{4-y} I_{a1} + a^{y-1} I_{a2} + I_{a0} \right) \qquad (16)$$

$$a^{4-z} V_{a1} + a^{z-1} V_{a2} + V_{a0} = 0 \qquad (17)$$

The exponents x, y and z are related as follows:

In the first sequence

$$y = x + 1 \qquad z = x + 2 \qquad (18)$$

In the second sequence

$$y = x + 2 \qquad z = x + 1 \qquad (19)$$

First Sequence

Substituting Eq. (18) and Eq. (3) in Eq. (16) and Eq. (17) and grouping terms give

$$a^{3-x} V_{a1} = a^{3-x} Z_f I_{a1} + a^x \left(Z_f + Z_2 \right) I_{a2} + \left(Z_f + Z_0 \right) I_{a0} \qquad (20)$$

$$a^{2-x} V_{a1} = a^{x+1} Z_2 I_{a2} + Z_0 I_{a0} \qquad (21)$$

Eqs. (15), (20), (21) may now be solved simultaneously for I_{a1}, I_{a2} and I_{a0}, giving

$$I_{a1} = \frac{Z_2 + Z_0 + Z_f}{Z_f Z_2 + Z_f Z_0 + Z_2 Z_0} = V_{a1} = \frac{V_{a1}}{Z} \qquad (22)$$

Where
$$Z = \frac{Z_f Z_2 + Z_f Z_0 + Z_2 Z_0}{Z_f + Z_2 + Z_0} \qquad (1)$$

$$I_{a2} = \frac{a^{1-2x}\, Z_f - a^{2-2x}\, Z_0}{Z_f Z_2 + Z_f Z_0 + Z_2 Z_0} V_{a1} \qquad (23)$$

$$I_{a0} = \frac{a^{2-x}\, Z_f - a^{1-x}\, Z_2}{Z_f Z_2 + Z_f Z_0 + Z_2 Z_0} V_{a1} \qquad (24)$$

If V_{a1} is eliminated among Eqs. (22), (23) and (24),

$$\frac{I_{a2}}{I_{a1}} = K_{o2} = \frac{a^{1-2x}\, Z_f - a^{2-2x}\, Z_0}{Z_f + Z_2 + Z_0}$$

$$\frac{I_{a0}}{I_{a1}} = K_{co} = \frac{a^{2-x}\, Z_f - a^{1-x}\, Z_2}{Z_f + Z_2 + Z_0} \qquad (25)$$

Second Sequence

If in the foregoing derivations Eq. (19) are used instead of Eq. (18) and the same steps are followed, Eq. (1) and the following equations are obtained.

$$\frac{I_{a2}}{I_{a1}} = K_{o2} = \frac{a^{3-2x}\, Z_f - a^{2-2x}\, Z_0}{Z_f + Z_2 + Z_0}$$

$$\frac{I_{a0}}{I_{a1}} = K_{c0} = \frac{a^{3-x}\, Z_f - a^{1-x}\, Z_2}{Z_f + Z_2 + Z_0} \qquad (26)$$

If $x = 1$, 2 and 3 in turn in Eqs. (25) and (26) the formulas of table 1 are obtained.

APPENDIX II

EQUATIONS DESCRIBING THE FAULT IN TERMS OF ALPHA BETA COMPONENTS AND CRISS CROSS COMPONENTS

<u>Fault b-c-n, Fault Impedance on Phase b</u>

Referring to Fig. 3a the following equations can be written

$$I_a = 0$$

$$V_c = 0 \qquad (27)$$

$$V_b = Z_f I_b$$

<u>Alpha Beta Components Approach</u> Eqs. (9), (10), and (27) give

$$I_\alpha + I_0 = \frac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) + \frac{1}{3}\left(I_a + I_b + I_c\right) = I_a$$

$$V_\alpha - 2V_0 = \frac{2}{3}\left(V_a - \frac{V_b + V_c}{2}\right) - \frac{2}{3}\left(V_a + V_b + V_c\right)$$

$$= - V_b - V_c = - Z_f I_b$$

$$= \frac{3}{2} Z_f I_\alpha - \frac{\sqrt{3}}{2} Z_f I_\beta$$

$$V_\beta = \frac{1}{\sqrt{3}}\left(V_b - V_c\right) = \frac{1}{\sqrt{3}} Z_f I_b$$

$$\frac{1}{2} Z_f I_\beta - \frac{\sqrt{3}}{2} Z_f I_\alpha$$

<u>Criss Cross Components Approach</u> Eqs. (11), (12) and (27) give

$$I_u + \frac{I_0}{\sqrt{2}} = \frac{1}{3\sqrt{2}}\left(2I_a - I_b - I_c\right) + \frac{1}{3\sqrt{2}}\left(I_a + I_b + I_c\right) = \frac{I_a}{\sqrt{2}}$$

$$= 0$$

$$V_u - \sqrt{2}\, V_0 = \frac{1}{3\sqrt{2}}\left(2V_a - V_b - V_c\right) - \frac{\sqrt{2}}{3}\left(V_a + V_b + V_c\right)$$

$$= -\frac{1}{\sqrt{2}}\left(V_b + V_c\right) = -\frac{1}{\sqrt{2}} Z_f I_b$$

$$= \frac{3}{2} Z_f I_u - \frac{\sqrt{3}}{2} Z_f I_v$$

$$V_v = \frac{1}{\sqrt{3}\sqrt{2}} (V_b - V_c) = \frac{1}{\sqrt{3}\sqrt{2}} Z_f I_b$$

$$= \frac{Z_f}{2} I_v - \frac{\sqrt{3}}{2} Z_f I_u$$

Fault a-b-n, Fault Impedance on Phase a

Referring to Fig. 3c the following equations can be written

$$I_c = 0$$

$$V_b = 0 \qquad (28)$$

$$V_a = Z_f I_a$$

Alpha Beta Components Approach Eqs. (9), (10) and (28) give

$$I_\alpha + \sqrt{3}\, I_\beta = \frac{2}{3} \left(I_a - \frac{I_b + I_c}{2} \right) + I_b - I_c = \frac{2}{3} (I_a + I_b)$$

$$= 2\, I_0 \qquad \text{(notice that } I_c = 0 \text{)}$$

$$V_\alpha - \frac{V_\beta}{\sqrt{3}} = \frac{2}{3} \left(V_a - \frac{V_b + V_c}{2} \right) - \frac{1}{3} (V_b - V_c) = \frac{2}{3} (V_a - V_b)$$

$$= \frac{2}{3} Z_f I_a$$

$$= Z_f I_\alpha + Z_f \frac{I_\beta}{\sqrt{3}}$$

$$V_\beta + \sqrt{3}\, V_0 = \frac{1}{\sqrt{3}} (V_b - V_c) + \frac{1}{\sqrt{3}} (V_a + V_b + V_c)$$

$$= \frac{1}{\sqrt{3}} V_a = \frac{1}{\sqrt{3}} Z_f I_a$$

$$= \frac{\sqrt{3}}{2} Z_f I_\alpha + \frac{Z_f}{2} I_\beta$$

<u>Alternative Solution</u> Eqs. (9), (10) and (28) give

$$I_\alpha + \sqrt{3}\, I_\beta = 2\, I_0 \qquad \text{(the same as above)}$$

$$2\, V_0 + \sqrt{3}\, V_\beta = \frac{2}{3}\left(V_a + V_b + V_c\right) + V_b - V_c = \frac{2}{3}\left(V_a - \frac{V_c}{2}\right)$$

$$= V_a \qquad \text{(notice that } V_b = 0\text{)}$$

$$\sqrt{3}\, V_\alpha - V_\beta = \frac{2}{\sqrt{3}}\left(V_a - \frac{V_b + V_c}{2}\right) - \frac{1}{\sqrt{3}}\left(V_b - V_c\right)$$

$$= \frac{2}{\sqrt{3}}\left(V_a - V_b\right) = \frac{2}{\sqrt{3}}\, Z_f\, I_a$$

$$= 2\, Z_f\left(\frac{I_\alpha}{\sqrt{3}} + \frac{I_0}{\sqrt{3}}\right)$$

<u>Criss Cross Components Approach</u> Eqs. (11), (12) and (28) give

$$I_u + \sqrt{3}\, I_v = \frac{1}{3\sqrt{2}}\left(2\, I_a - I_b - I_c\right) + \frac{1}{\sqrt{2}}\left(I_b - I_c\right)$$

$$= \frac{\sqrt{2}}{3}\left(I_a + I_b - 2\, I_c\right) = \frac{\sqrt{2}}{3}\left(I_a + I_b\right)$$

$$= \sqrt{2}\, I_0 \qquad \text{(notice that } I_c = 0\text{)}$$

$$V_u - \frac{V_v}{\sqrt{3}} = \frac{1}{3\sqrt{2}}\left(2\, V_a - V_b - V_c\right) - \frac{1}{3\sqrt{2}}\left(V_b - V_c\right)$$

$$= \frac{\sqrt{2}}{3}\left(V_a - V_b\right) = \frac{\sqrt{2}}{3}\, Z_f\, I_a$$

$$= Z_f\, I_u + Z_f\, \frac{I_v}{\sqrt{3}}$$

$$\frac{V_0}{\sqrt{2}} + \frac{V_v}{\sqrt{3}} = \frac{1}{3\sqrt{2}}\left(V_a + V_b + V_c\right) + \frac{1}{3\sqrt{2}}\left(V_b - V_c\right)$$

$$= \frac{1}{3\sqrt{2}}\left(V_a + 2\, V_b\right) = \frac{1}{3\sqrt{2}}\, Z_f\, I_a$$

$$= \frac{Z_f}{2}\, I_u + \frac{Z_f}{2}\, \frac{I_v}{\sqrt{3}}$$

Alternative Solution Eqs. (11), (12) and (28) give

$$I_u + \sqrt{3}\, I_v = \sqrt{2}\, I_0 \qquad \text{(the same as above)}$$

$$\sqrt{2}\, V_0 + \sqrt{3}\, V_v = \frac{\sqrt{2}}{3}\left(V_a + V_b + V_c\right) + \frac{1}{\sqrt{2}}\left(V_b - V_c\right)$$

$$= \frac{1}{3\sqrt{2}}\left(2\, V_a - V_c\right)$$

$$= V_u \qquad \text{(notice that } V_b = 0 \text{)}$$

$$\sqrt{3}\, V_u - V_v = \frac{1}{\sqrt{3}\sqrt{2}}\left(2\, V_a - V_b - V_c\right) - \frac{1}{\sqrt{3}\sqrt{2}}\left(V_b - V_c\right)$$

$$= \frac{\sqrt{2}}{\sqrt{3}}\, V_a = \frac{\sqrt{2}}{\sqrt{3}}\, Z_f\, I_a$$

$$= 2\, Z_f \left(\frac{I_u}{\sqrt{3}} + \frac{I_0}{\sqrt{2}\sqrt{3}}\right)$$

Fault a-b-n, Fault Impedance on phase b

Referring to Fig. 3f the following equations can be written.

$$I_c = 0$$

$$V_a = 0$$

$$V_b = Z_f\, I_b \qquad (29)$$

Alpha Beta Components Approach Eqs. (9), (10) and (29) give

$$I_\alpha + \sqrt{3}\, I_\beta = \frac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) + I_b - I_c + \frac{2}{3}\left(I_a + I_b\right)$$

$$= 2\, I_0 \qquad \text{(notice that } I_c = 0 \text{)}$$

$$V_\alpha + V_0 = \frac{2}{3}\left(V_a - \frac{V_b + V_c}{2}\right) + \frac{1}{3}\left(V_a + V_b + V_c\right) = V_a$$

$$= 0$$

$$\frac{V_\beta}{3} + V_0 = \frac{1}{3}\left(V_b - V_c\right) + \frac{1}{3}\left(V_a + V_b + V_c\right) = \frac{2}{3}\, V_b$$

$$= \frac{2}{3} Z_f I_b = \frac{2}{3} Z_f \sqrt{3} I_\beta \qquad \text{(notice that } I_c = 0 \text{)}$$

$$= \frac{2}{\sqrt{3}} Z_f I_\beta$$

Criss Cross Components Approach Eqs. (11), (12) and (20) give

$$I_u + \sqrt{3} I_v = \frac{1}{3\sqrt{2}} \left(2 I_a - I_b - I_c \right) + \frac{1}{\sqrt{2}} \left(I_b - I_c \right)$$

$$= \frac{\sqrt{2}}{3} \left(I_a + I_b \right)$$

$$= -\sqrt{2} I_c \qquad \text{(notice that } I_c = 0 \text{)}$$

$$V_u + \frac{V_0}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \left(2 V_a - V_b - V_c \right) + \frac{1}{3\sqrt{2}} \left(V_a + V_b + V_c \right) = \frac{V_a}{\sqrt{2}}$$

$$= 0$$

$$\frac{V_v}{\sqrt{3}} + \frac{V_0}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \left(V_b - V_c \right) + \frac{1}{3\sqrt{2}} \left(V_a + V_b + V_c \right)$$

$$= \frac{\sqrt{2}}{3} V_b = \frac{\sqrt{2}}{3} Z_f I_b$$

$$= \frac{\sqrt{2}}{3} Z_f \sqrt{3} \sqrt{2} I_v \qquad \text{(notice that } I_c = 0 \text{)}$$

$$= \frac{2}{\sqrt{3}} Z_f I_v$$

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